

### PROBLEM:

A linear-FM “chirp” signal is one that sweeps in frequency from  $\omega_1 = 2\pi f_1$  to  $\omega_2 = 2\pi f_2$  as time goes from  $t = 0$  to  $t = T_2$ . We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A \cos(\alpha t^2 + \beta t + \phi) \quad (1)$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument  $\psi(t)$  is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly*.

$$\omega_i(t) = \frac{d}{dt}\psi(t) \quad \text{radians/sec} \quad (2)$$

There are examples on the CD-ROM in the Chapter 3 demos.

- (a) For the linear-FM “chirp” in (1), determine formulas for the beginning instantaneous frequency ( $\omega_1$ ) and the ending instantaneous frequency ( $\omega_2$ ) in terms of  $\alpha$ ,  $\beta$  and  $T_2$ . For this problem, assume that the starting time of the “chirp” is  $t = 0$ .
- (b) For the “chirp” signal

$$x(t) = \Re \left\{ e^{j2\pi(30t^2 - 30t)} \right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

- (c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range  $0 \leq t \leq 1$  sec.