PROBLEM:

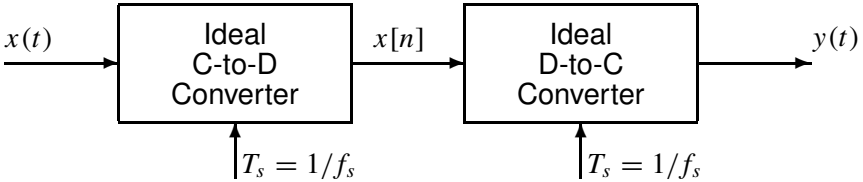

Figure 1: Ideal sampling and reconstruction system.

Shown in the figure above is an ideal C-to-D converter that samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. Suppose that $x(t)$ is given by

$$x(t) = [10 + 10\cos(500\pi t - \pi/2)]\cos(2000\pi t)$$

(a) Use Euler's formulas for the cosine functions to expand $x(t)$ in terms of complex exponential signals so that you can sketch the two-sided spectrum of this signal. Be sure to label important features of the plot. Is this waveform periodic? If so, what is the period?

(b) What is the *minimum* sampling rate f_s that can be used in the above system so that $y(t) = x(t)$?

(c) Plot the spectrum of the sampled signal $x[n]$ for the case when $f_s = 5000$. Your plot should include labels on the frequency (on the $\hat{\omega}$ scale), amplitude and phase of each spectrum component.