PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

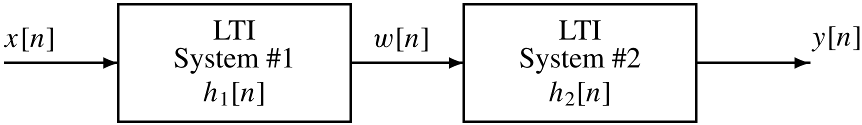

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that LTI System #1 is described by the difference equation

$$w[n] = x[n] - \alpha x[n-1].$$

Determine the impulse response $h_1[n]$ of the first system.

(b) The LTI System #2 is described by the impulse response

$$h_2[n] = \alpha^n(u[n] - u[n-L]) = \sum_{k=0}^{L-1} \alpha^k \delta[n-k] = \begin{cases} \alpha^n & n = 0, 1, \ldots, L-1 \\ 0 & \text{otherwise.} \end{cases}$$

For the special case of $L = 6$, use convolution to show that the impulse response sequence of the overall cascade system is

$$h[n] = h_1[n] * h_2[n] = \delta[n] - \alpha^6 \delta[n-6].$$

(c) Generalize your result in part (b) for the general case of L any integer value.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.

(e) Assuming that $0 < \alpha < 1$, how would you choose L so that $y[n] = x[n]$ in Figure 1; i.e., how would you choose L so that the second system "undoes" the effect of the first system?