PROBLEM:

The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 + e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}}). \tag{1}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint: Multiply out the factors to obtain a sum of powers of* $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n? *Hint: In this part, the answer is most obvious in the factored form of Eq. (1).*

(d) Use superposition to determine the output of this system when the input is

$$x[n] = 3 + \delta[n - 2] + \cos(0.5\pi n + \pi/4) \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results.