PROBLEM:

A periodic signal $x(t) = x(t + T_0)$ is described over one period $-T_0/2 \le t \le T_0/2$ by the equation

$$x(t) = \begin{cases} 10 & |t| < t_c \\ -2 & t_c < |t| \le T_0/2 \end{cases}$$

where $t_c < T_0/2$. In this problem assume that $T_0 = 5$ and $t_c = 1$.

(a) Sketch the periodic function $x(t)$ for t in the range $-T_0 < t < 2T_0$.

(b) Determine the D.C. coefficient X_0 using the parameters $T_0 = 5$ and $t_c = 1$.

(c) Determine the *fundamental frequency* ω_0 in the Fourier Series representation (rad/sec).

(d) Use the Fourier $\boxed{\textit{analysis}}$ integral(for $k \neq 0$)

$$X_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t} dt$$

to determine a general formula for the Fourier coefficients X_k in the representation

$$x(t) = X_0 + \Re e \left\{ \sum_{k=1}^{\infty} X_k e^{jk\omega_0 t} \right\}$$

Your final result could depend on t_c and T_0, but use $t_c = 1$ and $T_0 = 5$.
Note: The integral can be done over any period of the signal; in this case, the most convenient choice is from $-T_0/2$ to $T_0/2$.

(e) Sketch the spectrum of $x(t)$ for the case $t_c = 1$ and $T_0 = 5$. Include the DC component and also the first 2 non-zero frequency components in both positive and negative frequency. Label each component with its complex amplitude (magnitude and phase). Check your work by verifying that the conjugate property, $\frac{1}{2}X_{-k} = \frac{1}{2}X_k^*$, holds.
Note: When converting from $\Re e\{X\}$ to the spectrum, remember that $\Re e\{X\} = \frac{1}{2}X + \frac{1}{2}X^*$.