PROBLEM:

A linear time-invariant system has system function

$$H(z) = \sum_{k=0}^{3} (0.5)^k z^{-k}$$

(a) Write the difference equation relating the output $y[n]$ to the input $x[n]$.

(b) Determine the response of this system to a unit impulse input; i.e., find the output $y[n] = h[n]$ when the input is $x[n] = \delta[n]$. Plot $h[n]$ as a function of n.

(c) Find the poles and zeros of $H(z)$ and plot them in the complex z-plane.

Remember the formula $\sum_{k=0}^{L-1} \alpha^k = \dfrac{1 - \alpha^L}{1 - \alpha}$