PROBLEM:

Given a feedback filter defined via the recursion:

$$y[n] = y[n-1] - y[n-2] + x[n] \qquad \text{(DIFFERENCE EQUATION)} \qquad (1)$$

(a) When the input to the system is the impulse signal:

$$x[n] = \begin{cases} +1 & \text{when } n = 0 \\ 0 & \text{when } n \neq 0 \end{cases}$$

determine the output signal $y[n]$. Assume the "at rest" condition: i.e., the output signal is zero for $n < 0$. Since this is the impulse response, use the notation $h[n]$ for this output. It should be easy to generate a few values of $h[n]$ and then see that $h[n]$ is actually periodic for $n \geq 0$.

(b) Determine the frequency $\hat{\omega}_\circ$ of the signal $h[n]$ in part (a). In addition, write a formula for $h[n]$ in the form $A\cos(\hat{\omega}_\circ n + \phi)$ that is valid for $n \geq 0$.

(c) The z-transform operator representation for the system in (1) is

$$H(z) = \frac{1}{1 - z^{-1} + z^{-2}}$$

Find the roots of the denominator polynomial $A(z) = 1 - z^{-1} + z^{-2}$ and relate the angle of the root positions in the z-plane to the frequency of $h[n]$.