PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.9y[n-1] - 0.9x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} and as a ratio of polynomials in z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $H(e^{j\hat{\omega}})$, the frequency response of this system.

(d) Show that $|H(e^{j\hat{\omega}})|^2 = 1$ for all $\hat{\omega}$.

Answers to Problem 6.1:

(a) $H(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$

(b) Poles at $z = 0$ (2 of them); Zeros at $z = e^{\pm j(2\pi/3)}$

(c) $H(e^{j\hat{\omega}}) = \frac{1}{3}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) = \dfrac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)} e^{-j\hat{\omega}}$

(d) Sketch $|H(e^{j\hat{\omega}})| = \left| \dfrac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)} \right|$ and $\arg\{H(e^{j\hat{\omega}})\} = -\hat{\omega}$.

(e) $y[n] = 4 + 0.8047\cos[0.25\pi(n-2)]$