PROBLEM:

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

(a) Find the frequency response $\mathcal{H}(\hat{\omega})$, and then express it as a mathematical formula, in polar form (magnitude and phase).

(b) Plot the magnitude and phase of $\mathcal{H}(\hat{\omega})$ as a function of $\hat{\omega}$ for $-\pi < \hat{\omega} < \pi$. Do this by hand and with the MATLAB function `freqz`.

(c) Find all frequencies, $\hat{\omega}$, for which the response to the input $e^{j\hat{\omega}n}$ is zero.

(d) When the input to the system is $x[n] = \exp(j\pi n/11)$ determine the functional form for the output signal $y[n]$.

(e) *Impulse Response:* Determine the response of this system to a unit impulse input; i.e., find the output $y[n] = h[n]$ when the input is $x[n] = \delta[n]$. Plot $h[n]$ as a function of n.