PROBLEM:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

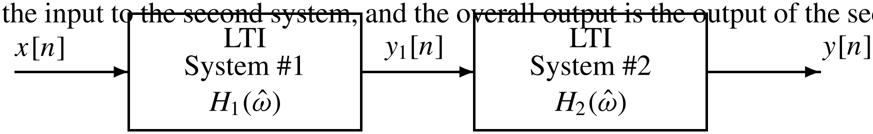

Figure 1: Cascade connection of two LTI systems.

(a) Show that if the input is $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, then the corresponding output of the overall system is

$$y[n] = H_2(\hat{\omega})H_1(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n} = \mathcal{H}(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$

where $H_1(\hat{\omega})$ is the frequency response of the first system and $H_2(\hat{\omega})$ is the frequency response of the second system. That is, show that the overall frequency response of a cascade of two LTI system is the product of the individual frequency responses, and therefore the cascade system is equivalent to a single system with frequency response $\mathcal{H}(\hat{\omega}) = H_2(\hat{\omega})H_1(\hat{\omega})$.

(b) Use the result of part (a) to show that the order of the systems is not important; i.e., show that for the same input $x[n]$ into the systems of Figs. 1 and 2, the overall outputs are the same ($w[n] = y[n]$).

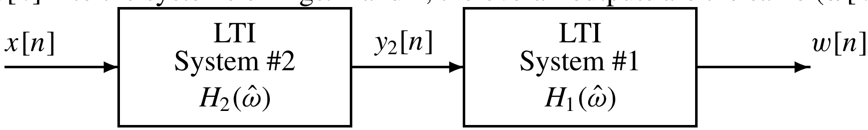

Figure 2: Equivalent system to system of Figure 1.

(c) Suppose that System #1 is described by the difference equation $y_1[n] = x[n] + x[n-2]$. and System #2 is described by the frequency response function $H_2(\hat{\omega}) = (1 - e^{-j\hat{\omega}2})$. Determine the frequency response function of the overall cascade system.

(d) Sketch the frequency response (magnitude and phase) of the overall cascade system for $-\pi \leq \hat{\omega} \leq \pi$.

(e) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1 and $w[n]$ to $x[n]$ in Fig. 2.