PROBLEM:

A linear time-invariant filter is described by the difference equation

$$y[n] = 0.8y[n-1] - 0.8x[n] + x[n-1]$$

(a) Determine the system function $H(z)$ for this system. Express $H(z)$ as a ratio of polynomials in z^{-1} (negative powers of z) and also as a ratio of polynomials in positive powers of z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, obtain an expression for $\mathcal{H}(\hat{\omega})$, the frequency response of this system.

(d) Show that $|\mathcal{H}(\hat{\omega})|^2 = 1$ for all $\hat{\omega}$.