PROBLEM:

Suppose that three systems are hooked together in "cascade." In other words, the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$, and the output of $\mathcal{S}_2$ is the input to $\mathcal{S}_3$. The three systems are specified as follows:

$$\mathcal{S}_1: \qquad y_1[n] = x_1[n] - x_1[n-1]$$

$$\mathcal{S}_2: \qquad y_2[n] = x_2[n] + x_2[n-2]$$

$$\mathcal{S}_3: \qquad y_3[n] = x_3[n-1] + x[n-2]$$

NOTE: the output of $\mathcal{S}_i$ is $y_i[n]$ and the input is $x_i[n]$.

Determine the equivalent system that is a single operation from the input $x[n]$ (into $\mathcal{S}_1$) to the output $y[n]$ which is the output of $\mathcal{S}_3$. Thus $x[n]$ is $x_1[n]$ and $y[n]$ is $y_3[n]$.

Write one difference equation that defines the overall system in terms of $x[n]$ and $y[n]$ only..