Pre-Lab and Warm-Up: You should read at least the Pre-Lab and Warm-up sections of this lab assignment and go over all exercises in the Pre-Lab section before going to your assigned lab session.

Verification: The Warm-up section of each lab must be completed during your assigned Lab time and the steps marked Instructor Verification must also be signed off during the lab time. One of the laboratory instructors must verify the appropriate steps by signing on the Instructor Verification line. When you have completed a step that requires verification, simply demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA when you leave the lab.

Lab Report: It is only necessary to turn in a report on Section 4 with graphs and explanations. You are asked to label the axes of your plots and include a title for every plot. In order to keep track of plots, include your plot inlined within your report. If you are unsure about what is expected, ask the TA who will grade your report.

1   Introduction

The objective of this lab is to introduce more complicated signals that are related to the basic sinusoid. These signals, which implement frequency modulation (FM) and amplitude modulation (AM), are widely used in communication systems such as radio and television, but they also can be used to create interesting sounds that mimic musical instruments. There are a number of demonstrations on the CD-ROM that provide examples of these signals for many different conditions.

2   Pre-Lab

We have spent a lot of time learning about the properties of sinusoidal waveforms of the form:

$$x(t) = A\cos(2\pi f_0 t + \phi) = \Re e\{A e^{j\phi} e^{j2\pi f_0 t}\} \tag{1}$$

In this lab, we will extend our treatment of sinusoidal waveforms to more complicated signals composed of sums of sinusoidal signals, or sinusoids with changing frequency.

2.1   Amplitude Modulation

If we add several sinusoids, each with a different frequency (f_k) we can express the result as:

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) = \Re e\{\sum_{k=1}^{N} (A_k e^{j\phi_k}) e^{j2\pi f_k t}\} \tag{2}$$

where $A_k e^{j\Phi k}$ is the complex amplitude of the kth complex exponential term. The choice of f_k will determine the nature of the signal—for amplitude modulation or beat signals we

pick two or three frequencies very close together. See Chapter 3 for a more detailed discussion of beat signals.

2.2 Frequency Modulated Signals

We will also look at signals in which the frequency varies as a function of time. In the constant-frequency sinusoid (1) the argument of the cosine is also the exponent of the complex exponential, so the angle of this signal is the exponent $(2\pi f_0 t + \Phi)$. This angle function changes linearly versus time, and its time derivative is $2\pi f_0$ which equals the constant frequency of the cosine in rad/sec.

A generalization is available if we adopt the following notation for the class of signals represented by a cosine function with a time-varying angle:

$$x(t) = A\cos(\Psi(t)) = \Re e\{Ae^{j\Psi(t)}\}$$

(3)

The time derivative of the angle from (3) gives a frequency in rad/sec

$$\omega_i(t) = \frac{d}{dt}\Psi(t) \qquad \text{(rad/sec)}$$

but we prefer units of hertz, so we divide by 2_ to define the instantaneous frequency:

$$f_i(t) = \frac{1}{2\Pi}\frac{d}{dt}\Psi(t) \qquad \text{(Hz)}$$

(4)

2.3 Chirp, or Linearly Swept Frequency

A chirp signal is a sinusoid whose frequency changes linearly from a starting value to an ending one. The formula for such a signal can be defined by creating a complex exponential signal with quadratic angle by defining $\Psi(t)$ in (3) as

$$\Psi(t) = 2\pi\mu t^2 + 2\pi f_0 t + \phi$$

The derivative of $\Psi(t)$ yields an instantaneous frequency (4) that changes linearly versus time.

$$f_i(t) = 2\mu t + f_0$$

The slope of $f_i(t)$ is equal to 2μ and its intercept is equal to f_0. If the signal starts at time t = 0 secs., then f0 is also the starting frequency. The frequency variation produced by such a time-varying angle is called frequency modulation. This kind of signal is an example of a frequency modulated (FM) signal. More generally, we often consider them to be part of a larger class called angle modulation signals. Finally, since the linear variation of the frequency can produce an audible sound similar to a siren or a chirp, the linear-FM signals are also called "chirps."

2.4 LabVIEW Synthesis of Chirp Signals

The following MathScript code will synthesize a chirp:

```
tt = 0 : dt : dur;
psi = 2*pi*(100 + 200*tt + 500*tt.*tt);
xx = real( 7.7*exp(j*psi) );
```

Place this code in a MathScript node, and build the following block diagram and front panel.
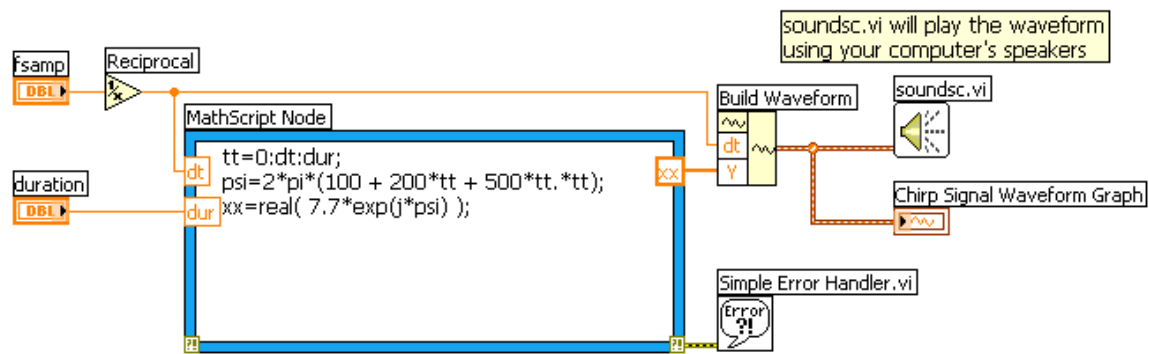
Be sure to connect the simple error handler to the MathScript node. This will tell you if there is an error in your MathScript code.

(a) Determine the total duration of the synthesized signal in seconds, and also the length of the tt vector (number of samples).

(b) In the MathScript node, signals can only be synthesized by evaluating the signal's defining formula at discrete instants of time. These are called samples of the signal. For the chirp we do the following:

$$x(t_n) = A\cos(2\pi\mu t_n^2 + 2\pi f_0 t_n + \phi)$$

where tn = nTs represents discrete time instants. In the MathScript node above, what is the value for tn? What are the values of A, μ, f0, and Φ?

(c) Determine the range of frequencies (in hertz) that will be synthesized by the VI above. Make a sketch by hand of the instantaneous frequency versus time. What are the minimum and maximum frequencies that will be heard?

(d) Listen to the signal to determine whether the signal's frequency content is increasing or decreasing.

3   Warm-up

3.1   Beat Control GUI

To assist you in your experiments with beat notes and AM signals, the tool called beatcon has been created. This user interface controller will exhibit the basic signal shapes for beat signals and play the signals. A small control panel will appear on the screen with buttons and sliders that vary the different parameters for the beat signals. Experiment with the control panel and use it to produce a beat signal with two frequency components at 850 Hz and 870 Hz. Demonstrate the plot and sound to your TA.

Instructor Verification (separate page)

3.2   Function for a Chirp

Use your code from the pre-lab as a starting point in order to write a VI that will synthesize a linear-FM "chirp" signal. Your VI should take the same inputs as the pre-lab, but you should add inputs for the starting and ending frequencies of the chirp signal. Name this VI "mychirp.vi".

As a test case, generate a chirp sound whose frequency starts at 2500 Hz and ends at 500 Hz; its duration should be 1.5 sec. Listen to the chirp using soundsc.vi, which is located in the DSP First palette.

| **Instructor Verification** (separate page) |
| --- |

3.3 Advanced Topic: Spectrograms

It is often useful to think of signals in terms of their spectra. A signal's spectrum is a representation of the frequencies present in the signal. For a constant frequency sinusoid as in (1) the spectrum consists of two components, one at $2\pi f_0$, the other at $-2\pi f_0$. For more complicated signals, the spectrum may be very interesting and, in the case of FM, the spectrum is considered to be time-varying. One way to represent the time-varying spectrum of a signal is the *spectrogram* (see Chapter 3 in the text). A spectrogram is found by estimating the frequency content in short sections of the signal. The magnitude of the spectrum over individual sections is plotted as intensity or color on a two-dimensional plot versus frequency and time.

| When unsure about a command, use help. |
| --- |

There are a few important things to know about spectrograms:
1. In LabVIEW, the VI specgram.vi in the DSP First palette will compute the spectrogram. Place this VI on the block diagram and open context help to learn more about it. Open context help by clicking on the VI and pressing CTRL+H.
2. Spectrograms are numerically calculated and only provide an estimate of the time-varying frequency content of a signal. There are theoretical limits on how well spectrograms can actually represent the frequency content of a signal. A later lab will treat this problem when we use the spectrogram to extract the frequencies of piano notes.
3. A common call to the VI might look like this:
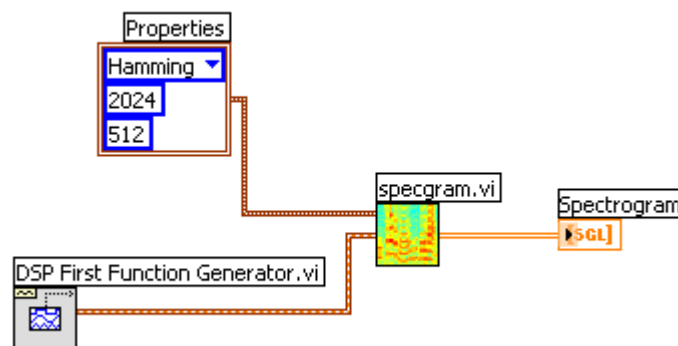
The properties input allows us to change the behavior of the sub-VI without writing a new function. The first property is the window type, the second is the window length and the third is the time increment. The second argument, the window length, can be varied to get different looking spectrograms. The spectrogram is able to "see" the separate spectrum lines with a longer window length, e.g., 1024 or 2048.

In order to see what the spectrogram produces, run the following code:
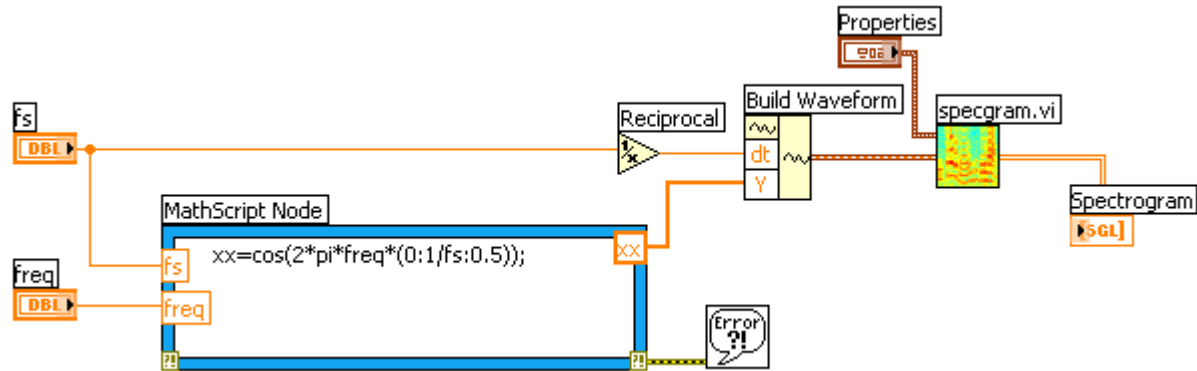
The front panel should look similar to this.
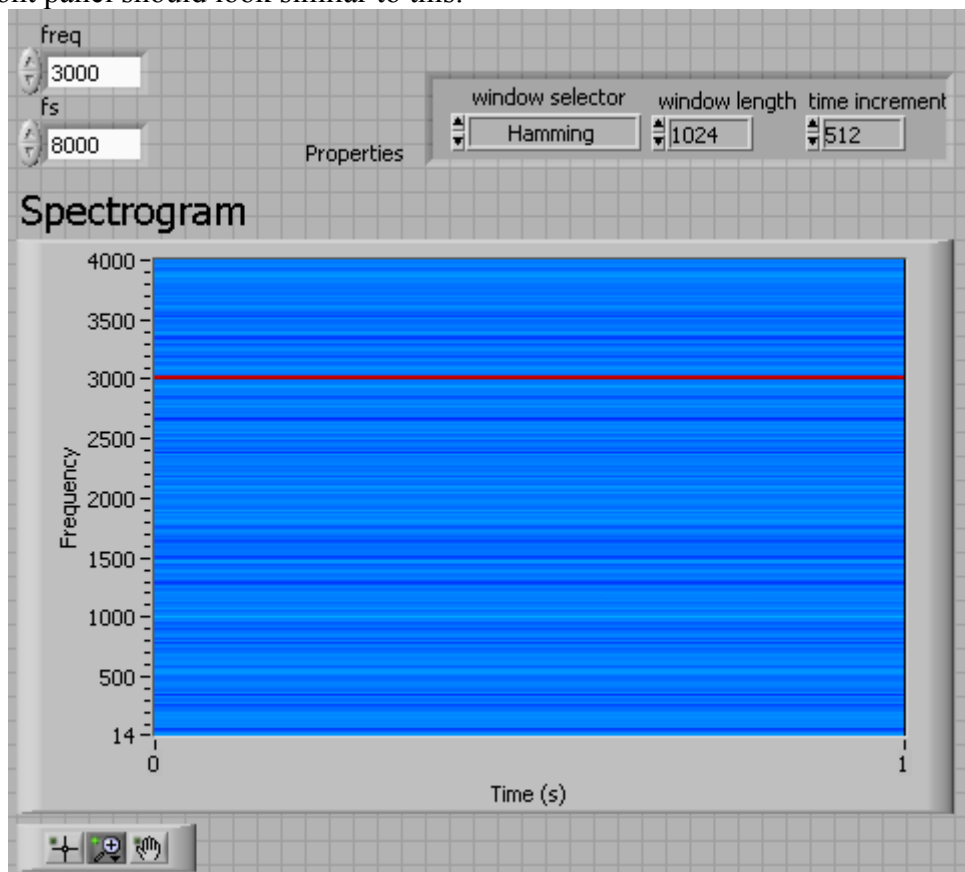
Notice that the spectrogram image contains one horizontal line at the correct frequency of the sinusoid.

4 Lab Exercise: Chirps and Beats

For the lab exercise and lab report, you will synthesize some AM and FM signals. In order to verify that they have the correct frequency content, you will use the spectrogram. Your lab report should discuss the connection between the "time-domain" definition of the signal and its "frequency-domain" content.
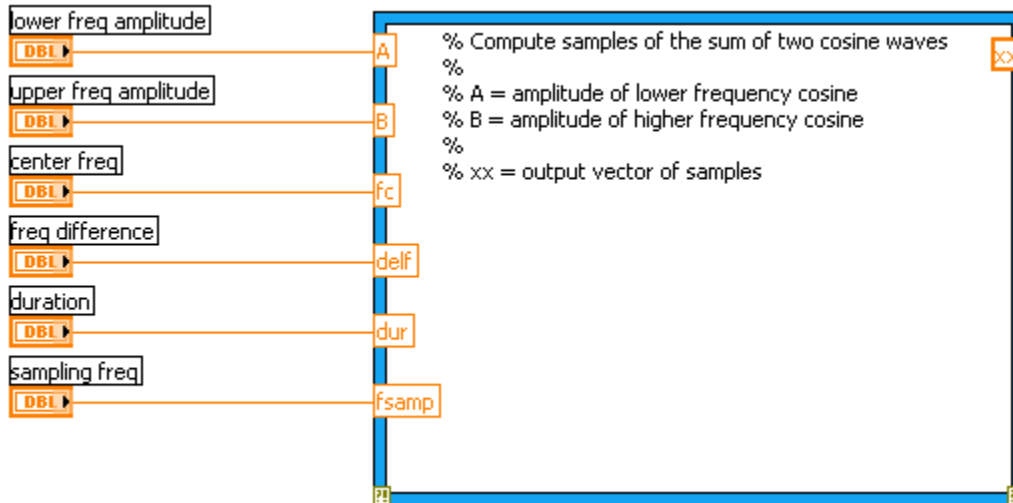
4.1 Beat Notes

In the section on beat notes in Chapter 3 of the text, we analyzed the situation in which we had two sinusoidal signals of slightly different frequencies; i.e.,

$$x(t) = A\cos(2\pi(f_c - f_\Delta)t) + B\cos(2\pi(f_c + f_\Delta)t) \qquad (5)$$

In this part, we will compute samples of such a signal and listen to the result.

(a) Write a VI that implements (5) using the following basic architecture. Remember you will have to add indicators and other functions outside the MathScript node to complete the exercise.

Include a copy of your VI in your lab report. You might want to call the sine_syn VI written in Lab 2 to do the calculation.

(b) To assist you in your experiments with beat notes, a tool called beatcon has been created. Use the values A=10, B=10, fc=1000, delf=10, fsamp=11025, and dur=1 secs. Plot the first 0.2 seconds of the resulting signal. Describe the waveform and explain its properties. Hand in a copy of your plot with measurements of the period of the "envelope" and period of the high frequency signal underneath the envelope.

(c) (Optional) Experiment with different values of the frequency difference f_Δ. Listen to the sounds and think about the relationship between the sound and waveform.

4.2 More on Spectrograms

Beat notes provide an interesting way to investigate the time-frequency characteristics of spectrograms. Although some of the mathematical details are beyond the reach of this course, it is not difficult to appreciate the following: There is a fundamental trade-off between knowing which frequencies are present in a signal (or its spectrum) and knowing how those frequencies vary with time. As mentioned previously in Section 3.3, a spectrogram estimates the frequency content over short sections of the signal. If we make the section length very short we can track rapid changes in the frequency. However, shorter sections lack the ability to do accurate frequency measurement because the amount of input data is limited. On the other hand, long sections can give excellent frequency measurements, but fail to track frequency changes well. For example, if a signal is the sum of two sinusoids whose frequencies are nearly the same, a long section length is needed to "resolve" the two sinusoidal components. This trade-off between the

section length (in time) and frequency resolution is equivalent to Heisenburg's Uncertainty Principle in physics.

When A =B in (5), the beat signal can be expressed as

$$x(t) = A\cos(2\pi(f_c - f_\Delta)t) + A\cos(2\pi(f_c + f_\Delta)t) = A[\cos(2\pi f_\Delta t)]\cos(2\pi f_c t)$$

Therefore, a beat note signal may be viewed as two signals with different constant frequencies, or as a single frequency signal whose amplitude varies with time. Both views will be useful in evaluating the effect of window length when finding the spectrogram of a beat signal.

      (a) Create and plot a beat signal with
          a. f_Δ = 32 Hz
          b. T_{dur} = 0.26 sec
          c. f_s = 11025 Hz
          d. f_c = 2000 Hz

      (b) Find the spectrogram using a window length of 2048 using the following code.
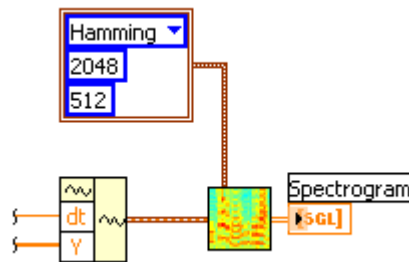
          Comment on what you see. Are the correct frequencies present in the spectrogram? If necessary, use the zoom tool to examine the important region of the spectrogram.

      (c) Find the spectrogram using a window length of 16. This can be accomplished by changing the window length in the properties constant from 2048 to 16. Comment on what you see, and compare to the previous spectrogram.

4.3 Spectrogram of a Chirp

Use the mychirp VI (written during the Warm-up) to synthesize a chirp signal for your lab report. Use the following parameters:

      1. A total time duration of 3 secs. with a D/A conversion rate of f_s = 11025 Hz.

      2. The instantaneous frequency starts at 5,000 Hz and ends at 300 Hz.

Listen to the signal. What comments can you make regarding the sound of the chirp (e.g., is the frequency movement linear)? Does it chirp down, or chirp up? Create a spectrogram of this chirp signal, and use it to verify that you have the correct instantaneous frequencies.

4.4 A Chirp Puzzle

Synthesize a second "chirp" signal (for your lab report) with the following parameters:

      1. A total time duration of 3 secs. with a sampling rate of f_s = 11025 Hz.

2. The instantaneous frequency starts at 3,000 Hz and ends at −2, 000 Hz (negative frequency).

Listen to the signal. Does it chirp down, or chirp up, or both?

Create a spectrogram of this second chirp signal.

Use the theory of the spectrum (with its positive and negative frequency components) to help explain what you hear and what you see in the spectrogram. In other words, the changing instantaneous frequency implies that the frequency components in the spectrum are moving.

Lab 03
INSTRUCTOR VERIFICATION SHEET

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: _____   Date of Lab:_____

Part 3.1 Demonstrate usage of the Beat Control GUI.

   Verified: _____   Date/Time:_____

Part 3.2 Demonstrate the mychirp.vi function. In the space below write how you would call the function with a correct set of arguments.
   Verified: _____   Date/Time:_____