Pre-Lab and Warm-Up: You should read at least the Pre-Lab and Warm-up sections of this lab assignment and go over all exercises in the Pre-Lab section before going to your assigned lab session.

Verification: The Warm-up section of each lab must be completed **during your assigned Lab time** and the steps marked *Instructor Verification* must also be signed off **during the lab time**. One of the laboratory instructors must verify the appropriate steps by signing on the **Instructor Verification** line. When you have completed a step that requires verification, simply demonstrate the step to the TA or instructor. Turn in the completed verification sheet to your TA when you leave the lab.

Lab Report: It is only necessary to turn in a report on Section 3 with graphs and explanations. You are asked to **label** the axes of your plots and include a title for every plot. In order to keep track of plots, include your plot *inlined* within your report. If you are unsure about what is expected, ask the TA who will grade your report.

# 1 Pre-Lab

The goal of this lab is to study the response of FIR filters to inputs such as complex exponentials and sinusoids. In the experiments of this lab, you will use the filter VI to implement filters and freqz.vi to obtain the filter's frequency response. Both are located in the DSP First Toolkit. As a result, you should learn how to characterize a filter by knowing how it reacts to different frequency components in the input.

This lab also introduces two practical filters: bandpass filters and nulling filters. Bandpass filters can be used to detect and extract information from sinusoidal signals, e.g., tones in a touch-tone telephone dialer. Nulling filters can be used to remove sinusoidal interference, e.g., jamming signals in radar.

## 1.1 Frequency Response of FIR Filters

The output or *response* of a filter for a complex sinusoid input, $e^{j\hat{\omega}n}$, depends on the frequency, $\hat{\omega}$. Often a filter is described solely by how it affects different input frequencies—this is called the *frequency response*.

For example, the frequency response of the two-point averaging filter

$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$$

can be found by using a general complex exponential as an input and observing the output or response.

$$x[n] = Ae^{j(\hat{\omega}n+\phi)} \tag{1}$$

$$y[n] = \frac{1}{2}Ae^{j(\hat{\omega}n+\phi)} + \frac{1}{2}Ae^{j(\hat{\omega}(n-1)+\phi)} \tag{2}$$

$$= Ae^{j(\hat{\omega}n+\phi)}\frac{1}{2}\{1 + e^{-j\hat{\omega}}\} = Ae^{j(\hat{\omega}n+\phi)} \cdot H(e^{j\hat{\omega}}) \tag{3}$$

In (3) there are two terms, the original input, and a term that is a function of $\hat{\omega}$. This second term is the frequency response and it is commonly denoted by H($e^{j\hat{\omega}}$), which in this case is

$$H(e^{j\hat{\omega}}) = \frac{1}{2}\{1 + e^{-j\hat{\omega}}\} \tag{4}$$

Once the frequency response, H($e^{j\hat{\omega}}$), has been determined, the effect of the filter on any complex exponential may be determined by evaluating H($e^{j\hat{\omega}}$) at the corresponding frequency. The output signal y[n], will be a complex exponential whose complex amplitude has a constant magnitude and phase. The phase describes the phase change of the complex sinusoid and the magnitude describes the gain applied to the complex sinusoid.

The frequency response of a general FIR linear time-invariant system is

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} \tag{5}$$

In the example above, M = 1, and $b_0 = \frac{1}{2}$ and $b_1 = \frac{1}{2}$ .

## 1.1.1 LabVIEW Function for Frequency Response

The DSP First Toolkit has a function called freqz.vi for computing the frequency response of a discrete-time LTI system. The following LabVIEW block diagram and front panel show how to use freqz to compute and plot both the magnitude (absolute value) and the phase of the frequency response of a two-point averaging system as a function of ω in the range $-\pi \le \hat{\omega} \le \pi$:
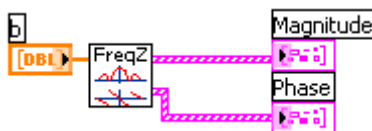


The frequency range for this VI is $-\pi \le \hat{\omega} \le \pi$ by default. Note: we will always use capital HH for the frequency response in MathScript nodes.

## 1.2 Periodicity of the Frequency Response

The frequency responses of discrete-time filters are *always* periodic with period equal to $2\pi$. Explain why this is the case by stating a definition of the frequency response and then considering two input sinusoids whose frequencies are $\hat{\omega}$ and $\hat{\omega} + 2\pi$.

$$x_1[n] = e^{j\hat{\omega}n} \qquad \text{versus} \qquad x_2[n] = e^{j(\hat{\omega}+2\pi)n}$$

Consult Chapter 6 for a mathematical proof that the outputs from each of these signals will be identical (basically because x1[n] is equal to x2[n].)

**The implication of periodicity is that a plot of H($e^{j\hat{\omega}}$) only needs to extend over the interval $-\pi \le \hat{\omega} \le \pi$ or any other interval of length $2\pi$.**

## 1.3 Frequency Response of the Four-Point Averager

In Chapter 6 we examined filters that average input samples over a certain interval. These filters are called "running average" filters or "averagers" and they have the following form for the L-point averager:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] \tag{6}$$

(a) Use Euler's formula and complex number manipulations to show that the frequency response for the 4-point running average operator is given by:

$$H(e^{j\hat{\omega}}) = \frac{2\cos(0.5\hat{\omega}) + 2\cos(1.5\hat{\omega})}{4} e^{-j1.5\hat{\omega}} \tag{7}$$

(b) Implement equation 7 directly in LabVIEW. Use a vector that includes 400 samples between $-\pi$ and $\pi$ for ŵ. Since the frequency response is a complex-valued quantity, use abs and angle to extract the magnitude and phase of the frequency response for plotting. Plotting the real and imaginary parts of H(e^{jŵ}) is not very informative.

(c) In this part, use freqz.vi to compute H(e^{jŵ}) numerically (from the filter coefficients) and plot its magnitude and phase versus $\hat{\omega}$. Write the appropriate code to plot both the magnitude and phase of H(e^{jŵ}). Follow the example in Section 1.1.1. The filter coefficient vector for the 4-point averager is defined via:

bb = 1/4*ones(1,4);

Note: the freqz function evaluates the frequency response for all frequencies in the input waveform. It uses the summation in (5), not the formula in (7). The filter coefficients are defined in the assignment to filter coefficient vector. How do your results compare with part (b)?

## 1.4 Finding Elements in an Array

Often signal processing functions are performed in order to extract information that can be used to make a decision. The decision process inevitably requires logical tests, which might be done with if-then constructs. However, MathScript permits vectorization of such tests, and the find function is one way to do lots of tests at once. In the following example, find extracts all the numbers that "round" to 3:

$$xx = 1.4{:}0.33{:}5, \; jkl = find(round(xx){==}3), \; xx(jkl)$$

The argument of the find function can be any logical expression. Notice that find returns a list of indices where the logical condition is true.

This can also be done using "search all in 1D array.vi" from the DSP First Toolkit.  The following block diagram shows exactly how to perform the same operation as the find function above.  The search VI also includes an output indicating how many matches were found.
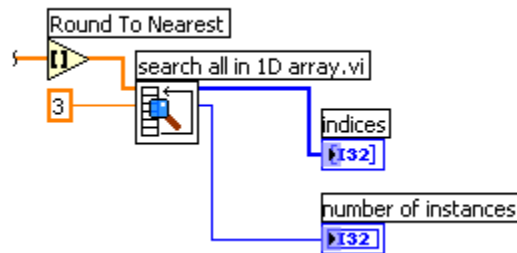


Now, suppose that you have a frequency response:
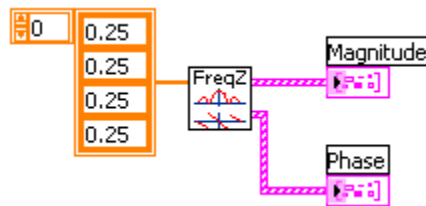


Now you can use the LabVIEW to determine the indices where the frequency response is zero, and then use those indices to display the list of frequencies where the response is zero. Since there might be round-off error in calculating frequency response, the logical test should probably be a test for those indices where the magnitude (absolute value) is less than some rather small number, e.g., $1 \times 10^{-6}$. You may have to experiment with this threshold number, however, in order to get proper results.  For this test, you can use the "conditional" input of the search all in 1D array VI to select "less than".  With this option selected, the VI will return all indices where the array element is less than the element input.  Remember that the Magnitude output is a 1D array containing a cluster of two elements, the X and Y axes.  Therefore, you must use "separate freqz output.vi" to break out the X and Y components of the response. Compare your answer to the frequency response that you plotted for the four-point averager in Section 1.3.

# 2 Warm-up

The first objective of this warm-up is to use an interactive LabVIEW demo to demonstrate nulling. This demo, "dltidemo.vi", is on the DSP First CD-ROM under Chapter 6 demos.

## 2.1 LTI Frequency Response Demo
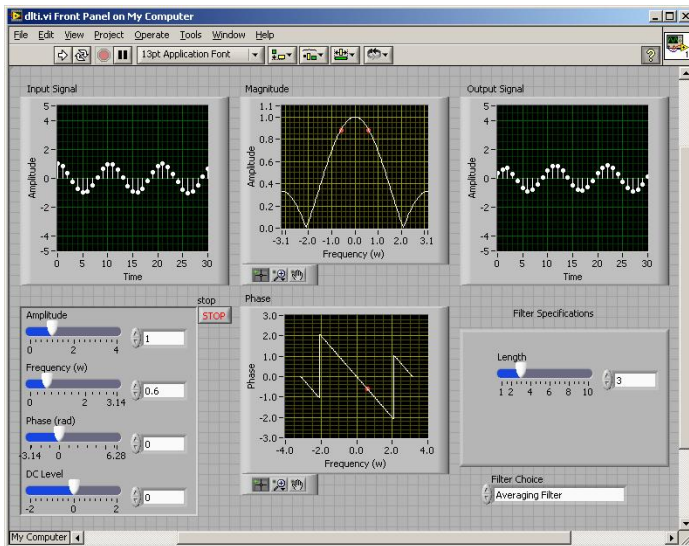


## 2.2 Cascading Two Systems

More complicated systems are often made up from simple building blocks. In Fig. 2, two FIR filters are shown connected "in cascade."
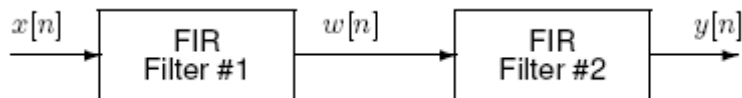


Figure 2: Cascade of two FIR filters

Assume that the system in Fig. 2 is described by the two equations:

$$w[n] = \sum_{\ell=0}^{M} \alpha^{\ell} x[n-\ell] \qquad \text{FIR Filter \#1}$$

$$y[n] = w[n] - \alpha w[n-1] \qquad \text{FIR Filter \#2}$$

(a) Use freqz.vi to get the frequency responses for the case where $\alpha = 0.8$ and $M = 9$. Plot the magnitude and phase of the frequency response for Filter #1, and also for Filter #2. Which one of these filters is a *lowpass filter*?

(b) Plot the magnitude and phase of the frequency response of the overall cascaded system.

(c) Explain how the individual frequency responses in part(a) are combined to get the overall frequency response in part(b). Comment on the magnitude combinations as well as the phase combinations.

<div style="border:1px solid black; text-align:center;">
**Instructor Verification** (separate page)
</div>

## 2.3 Deconvolution

In Lab 7, the two filters from Section 2.2 were used in an image deblurring experiment. You should now re-interpret how that experiment worked by explaining what happens *in the frequency domain*.

(a) If a single filter has a frequency response H($e^{j\hat{\omega}}$)= 1, how is the output of the filter y[n] related to the input x[n]?

(b) Ideally, a "deconvolved" output should look exactly like the input prior to blurring. If Filter #1 (in Fig. 2) has a frequency response H1($e^{j\hat{\omega}}$), and Filter #2 is H2($e^{j\hat{\omega}}$), explain why the condition H1($e^{j\hat{\omega}}$)H2($e^{j\hat{\omega}}$) = 1 will guarantee "perfectly deconvolution."

(c) The filters in Section 2.2 do not perform a perfect deconvolution (for the case α = 0.8 and M = 9).Use the frequency response from Section 2.2(b) to explain deviations from a perfect result.

<div style="border:1px solid black; text-align:center;">
Instructor Verification (separate page)
</div>

# 3 Lab Exercises

## 3.1 Nulling Filters for Rejection

Nulling filters are filters that completely eliminate some frequency component. If the frequency is $\hat{\omega} = 0$ or $\hat{\omega} = \pi$, then a two-point FIR filter will do the nulling. The simplest possible general nulling filter can have as few as three coefficients. If $\hat{\omega}_n$ is the desired nulling frequency, then the following length-3 FIR filter

$$y[n] = x[n] - 2\cos(\hat{\omega}_n)x[n-1] + x[n-2] \qquad (8)$$

will have a zero in its frequency response at $\hat{\omega} = \hat{\omega}_n$. For example, a filter designed to completely eliminate signals of the form $Ae^{j0.5\pi n}$ would have the following coefficients because we would pick the desired nulling frequency to be $\hat{\omega}_n = 0.5\pi$.

$$b_0 = 1, \qquad b_1 = -2\cos(0.5\pi) = 0, \; b_2 = 1.$$

(a) Design a filtering system that consists of the *cascade of two FIR nulling filters* that will eliminate the following input frequencies: $\hat{\omega} = 0.44\pi$, and $\hat{\omega} = 0.7\pi$. For this part, derive the filter coefficients of both nulling filters.

(b) Generate an input signal x[n] that is the sum of three sinusoids:

$$x[n] = 5\cos(0.3\pi n) + 22\cos(0.44\pi n - \frac{\pi}{3}) + 22\cos(0.7\pi n - \frac{\pi}{4})$$

Make the input signal 150 samples long over the range $0 \le n \le 149$.

(c) Use the FIR filter Vis or convultion to filter the sum of three sinusoids signal x[n] through the filters designed in part (a). Show the code that you wrote to implement the cascade of two FIR filters.

(d) Make a plot of the output signal—show the first 40 points. Determine (by hand) the exact mathematical formula (magnitude, phase and frequency) for the output signal for $n \ge 5$.

(e) Plot the mathematical formula determined in (d) to show that it matches the filter output from the FIR filter VI over the range $5 \le n \le 40$.

(f) Explain why the output signal is different for the first few points. How many "start-up" points are found, and how is this number related to the lengths of the filters designed in part (a)? Hint: consider the length of a single FIR filter that is equivalent to the cascade of two length-3 FIRs.

## *3.2 Simple Bandpass Filter Design*

The L-point averaging filter is a lowpass filter. Its passband width is controlled by L, being inversely proportional to L. In fact, you can use the GUI dltidemo to view the frequency response for different averagers and measure the passband widths. It is also possible to create a filter whose passband is centered around some frequency other than zero. One simple way to do this is to define the impulse response of an L-point FIR as:

$$h[n] = \frac{2}{L}\cos(\hat{\omega}_c n), \qquad 0 \le n < L$$

where L is the filter length, and $\hat{\omega}_c$ is the center frequency that defines the frequency location of the passband. For example, we would pick $\hat{\omega}_c = 0.44\pi$ if we want the peak of the filter's passband to be centered at $0.44\pi$. The bandwidth of the bandpass filter is controlled by L; the larger the value of L, the narrower the bandwidth. This particular filter is also discussed in the section on useful filters in Chapter 7 of the text.

(a) Generate a bandpass filter that will pass a frequency component at $\hat{\omega} = 0.44\pi$. Make the filter length (L) equal to 10. Since we are going to be filtering the signal defined in

section 3.1(b), measure the gain of the filter at the three frequencies of interest: $\hat{\omega} = 0.3\pi$, $\hat{\omega} = 0.44\pi$ and $\hat{\omega} = 0.7\pi$.

(b) The *passband* of the BPF filter is defined by the region of the frequency response where $|H(e^{j\hat{\omega}})|$ is close to its maximum value. If we define the maximum to be Hmax, then the passband width is defined as the length of the frequency region where the ratio $|H(e^{j\hat{\omega}})|$/Hmax is greater than $1/\sqrt{2} = 0.707$. Figure 3 shows how to define the passband and stopband. Note: you can use MATLAB's find function to locate those frequencies where the magnitude satisfies $|H(e^{j\hat{\omega}})| \geq 0.707$Hmax.

The *stopband* of the BPF filter is defined by the region of the frequency response where $|H(e^{j\hat{\omega}})|$ is close to zero. In this case, we will define the stopband as the region where $|H(e^{j\hat{\omega}})|$ is less than 25% of the maximum.

Make a plot of the frequency response for the L = 10 bandpass filter from part (a), and determine the passband width (at the 0.707 level). Repeat the plot for L = 20 and L = 40, so you can explain how the width of the passband is related to filter length L, i.e., what happens when L is doubled or halved.
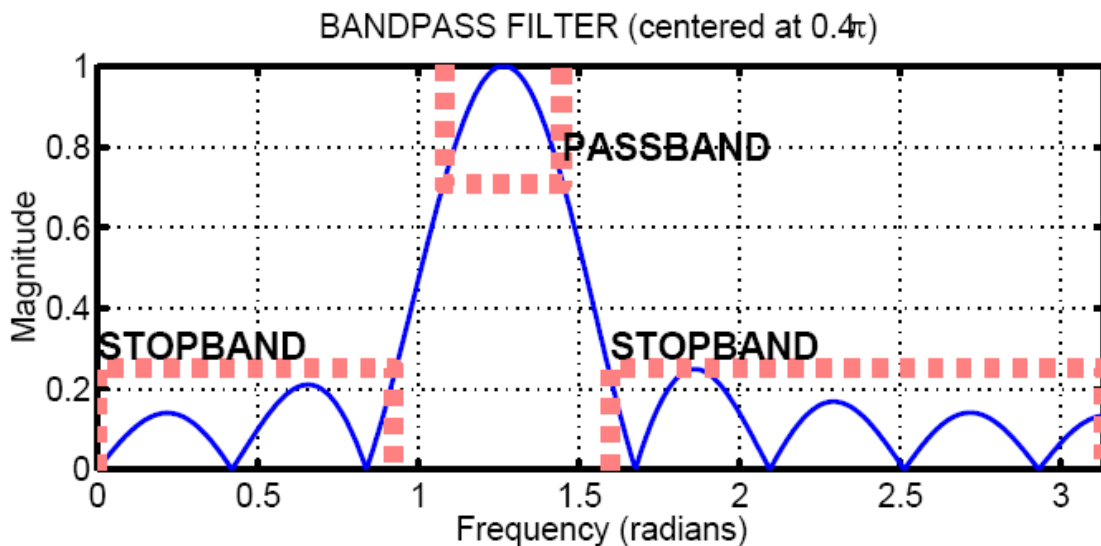


Figure 3: Passband and Stopband for a typical FIR bandpass filter. In this case, the maximum value is 1, the passband is the region where the frequency response is greater than $1/\sqrt{2} = 0.707$, and the stopband is defined as the region where the frequency response is less than 25% of the maximum.

(c) Comment on the selectivity of the L = 10 bandpass filter. In other words, which frequencies are "passed by the filter?" Use the frequency response to explain how the filter can pass one component at $\hat{\omega} = 0.44\pi$, while reducing or rejecting the others at $\hat{\omega}$ $0.3\pi$ and $\hat{\omega} = 0.7\pi$.

(d) Generate a bandpass filter that will pass the frequency component at $\hat{\omega} = 0.44\pi$, but now make the filter length (L) long enough so that it will also greatly reduce frequency

components at (or near) $\hat{\omega} = 0.3\pi$ and $\hat{\omega} = 0.7\pi$. Determine the smallest value of L so that

•  Any frequency component satisfying $|\hat{\omega}| \leq 0.3\pi$ will be reduced by a factor of 10 or more.

•  Any frequency component satisfying $0.7\pi \leq |\hat{\omega}| \leq \pi$ will be reduced by a factor of 10 or more.

This can be done by making the passband width very small.

(e) Use the filter from the previous part to filter the "sum of 3 sinusoids" signal from Section 3.1. Make a plot of 100 points of the input and output signals, and explain how the filter has reduced or removed two of the three sinusoidal components.

(f) Make a plot of the frequency response (magnitude only) for the filter from part (d), and explain how $H(e^{j\hat{\omega}})$ can be used to determine the relative size of each sinusoidal component in the output signal. In other words, connect a mathematical description of the output signal to the values that can be obtained from the frequency response plot.

# Lab 08
# INSTRUCTOR VERIFICATION SHEET

For each verification, be prepared to explain your answer and respond to other related questions that the lab TA's or professors might ask. Turn this page in at the end of your lab period.

Name: _____          Date of Lab:_____

Part 2.1(d) Use the dltidemo to illustrate the operation of a 9-point averaging filter. Determine the amount of delay through the filter, and write your answer in the space below.

Verified: _____          Date/Time:_____

Part 2.1(f) Use the dltidemo to find a digital FIR filter that will null the input signal. Determine the filter length, and write your answer in the space below.

Verified: _____          Date/Time:_____

Part 2.2 Plot the frequency response of the two filters in the cascade combination, and then explain how the magnitudes are combined and how the phases are combined to get the overall filter. Check the range of frequencies (ŵ) used for the plot.

Verified: _____          Date/Time:_____

Part 2.3 Explain how close the frequency response of the cascaded system is to the desired frequency response for "ideal deconvolution."

Verified: _____          Date/Time:_____