Lab P-16: PeZ - The z, n, and $\hat{\omega}$ Domains

The lab report/verification will be done by filling in the last page of this handout which addresses a list of observations to be made when using the `PeZ` GUI.

1 Introduction & Objective

The objective for this lab is to build an intuitive understanding of the relationship between the location of poles and zeros in the z-domain, the impulse response $h[n]$ in the n-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the $\hat{\omega}$-domain). A graphical user interface (GUI) called `PeZ` was written in MATLAB for doing interactive explorations of the three domains.[1] `PeZ` is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G\frac{\displaystyle\prod_{k=1}^{M}(1 - z_k z^{-1})}{\displaystyle\prod_{k=1}^{N}(1 - p_k z^{-1})} = \frac{\displaystyle\sum_{k=0}^{M}b_k z^{-1}}{1 - \displaystyle\sum_{k=1}^{N}a_k z^{-1}}$$

2 Warm-up

Invoke `PeZ` from the *DSP First* MATLAB *Toolbox*, which can be downloaded from *DSP First companion website*. A control panel with a few buttons and a plot of the unit circle in the complex z-plane will pop up. You can view a tutorial from the Demos link. Here are a few things to try. You can use the control window to selectively place poles and zeros in the z-plane, and then observe (in a second window) how their placement affects the impulse and frequency responses.[2] If the plots need manual updating, click on the `Redo Plots` button under the `<Quicksize...>` menu. The `Real Time Drag Plots` button will put `PeZ` in a mode such that an individual pole/zero (pair) can be moved around and the corresponding $H(e^{j\hat{\omega}})$ and $h[n]$ plots will be updated as you drag the pole (or zero).

Since exact placement of poles and zeros with the mouse is difficult, an `Edit By Co-Ord` button is provided for numerical entry of the real and imaginary parts, or magnitude and angle (a separate edit window appears when you use this option). Before you can edit a pole or zero, however, you must first select it with the mouse. Removal of individual poles or zeros can also be performed by clicking on the `Delete Poles & Zeros` button (again, a separate window will appear). Note that all poles and/or zeros can be easily cleared by clicking on the `<Clear...>` menu, and then selecting `Poles`, `Zeros`, or `All`.

Play around with `PeZ` for a few minutes to gain some familiarity with the interface. Implement the following first-order system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its poles and zeros at the correct location in the z-plane. First try placing the pole and zero with the mouse, and then use the Edit by Co-Ord feature.

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[1] `PeZ` was written originally by Craig Ulmer, and is now maintained by Greg Krudysz.

[2] There is a PeZ Tutorial on the *DSP First companion website*.
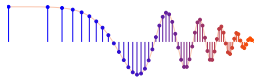
3   Relationships between z, n, and $\hat{\omega}$ domains

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

- How does $h[n]$ change with respect to its rate of decay? For example, when $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when a is smaller.

- If $h[n]$ exhibits an oscillation period, what is the period? Also, estimate the decay rate of the "envelope."

- How does $H(e^{j\hat{\omega}})$ change with respect to peak location and peak width?

Among the demos for Chapters 7 and 8 on the *DSP First companion website*, there are ones entitled *Three-Domains: FIR* and *Three-Domains: IIR* which provide examples of these relationships.

4   Lab Exercise: Real Poles

(a) Use PeZ to place a single pole at $z = \frac{1}{2}$. You may have to use the `Edit by Co-Ord` button to get the location exactly right. Use the plots for this case as the reference for answering the next four parts.

(b) Move the pole close to the origin (still on the real axis). You can do this by clicking on the pole and dragging it to the new location. Describe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$.

(c) You can also move poles and zeros under the influence of the `Real Time Drag Plots` option in PeZ. When this box is checked, the impulse response and frequency response plots are updated while you move the pole (or zero). Once this mode is set, click on the pole you want to move and start to drag it slowly. Watch for the update of the plots in the secondary window. After the real-time updating has started, you can release the mouse button and the pole (or zero) will follow the cursor. Click on the pole once more to stop moving it and to stop the updating. It is sometimes a little tricky to use this feature. Also the display may be jerky unless you have a high-performance computer with fast graphics.

   Move the real pole slowly from $z = \frac{1}{2}$ to $z = 1$ and observe the changes in the impulse response $h[n]$ and the frequency response $H(e^{j\hat{\omega}})$.

(d) Place the pole exactly on the unit circle (or maybe just inside at a radius of 0.99999999). Describe the changes in $h[n]$ and $H(e^{j\hat{\omega}})$. What do you expect to see for $H(e^{j\hat{\omega}})$?

(e) Move the pole outside the unit circle. Describe the changes in $h[n]$. Explain how the appearance of $h[n]$ validates the statement that the system is not stable. In this case, the frequency response $H(e^{j\hat{\omega}})$ is not legitimate because the system is no longer stable.

(f) In general, where should poles be placed to guarantee system stability? By stability we mean that the system's output does not blow up.

5   Lab Exercise: Complex Poles and Zeros

PeZ assumes real coefficients for the numerator and denominator polynomials. Therefore, if we enter a complex pole or zero, PeZ will automatically insert second root at the conjugate location. For example, if we place a root at $z = \frac{1}{3} + j\frac{1}{2}$, then we will also get one at $z = \frac{1}{3} - j\frac{1}{2}$.
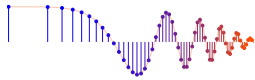
(a) What property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ will guarantee that the roots come in conjugate pairs?

(b) Clear all the poles and zeros from PeZ. Now place a pole with magnitude 0.85 at an angle of 45°; and then two zeros at the origin. Note that PeZ automatically places a conjugate pole in the z-domain. The frequency response has a peak—record the frequency (location) of this peak.

(c) Change the angle of the pole: move the pole to 90°, then 135°. Describe the changes in $|H(e^{j\hat{\omega}})|$. Concentrate on the location of the peak.

Next, we will put complex zeros on the unit circle to see the effect on $|H(e^{j\hat{\omega}})|$.

(d) Clear all poles and zeros from PeZ. Now place zeros at the following locations: $z_1 = -1$, $z_2 = 0 - j$ and $z_3 = 0 + j$ (remember that conjugate pairs such as z_2 and z_3 will be entered simultaneously). Judging from the impulse and frequency responses what type of filter have you just implemented?

6 Lab Exercise: Filter Design

In this section, we will use PeZ to place the poles and zeros of $H(z)$ to make a filter with a desirable frequency response. *Filter design* is a process that selects the coefficients $\{a_k\}$ and $\{b_k\}$ to accomplish a
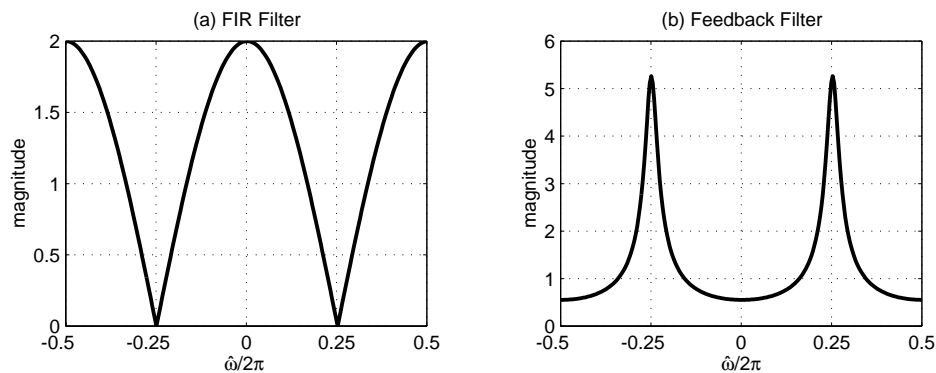

Figure 1: Magnitude response of two unknown filters. Frequency axis is normalized ($\hat{\omega}/2\pi$). Use PeZ to help you find the filter coefficients that will match these frequency responses as closely as possible. (a) Second-order FIR filter. (b) Second-order IIR filter.

given task. The task here is to create a filter that has a very narrow "notch." This filter would be useful for removing one frequency component while leaving others undisturbed. The *notch filter* can be synthesized from the cascade of two simpler filters shown in Fig. 1.

(a) Start the process by using PeZ to design each of the filters given in Fig. 1. (You will have to determine the locations of the poles and zeros from the plots in Fig. 1.) Both filters are second-order. Make sure that you enter the poles and zeros precisely. PeZ will do the conversion between between root locations and polynomial coefficients, but you could also do this with the MATLAB commands roots and poly. You can check your results by also calculating the filter coefficients by hand (see the next section on polynomials with complex coefficients). Record the coefficients of your filters in the table provided.
Note: Use PeZ or freqz() to verify that the frequency response of each filter is correct.
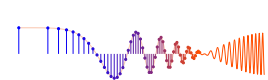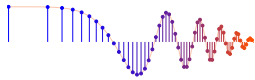
(b) Now use `PeZ` to connect the filters together in a cascade. Place the poles and zeros, and then view the frequency response. Determine the filter coefficients for the overall cascaded filter $H(z)$.

(c) Use `freqz()` to determine the frequency response of the cascade of the two filters that you "designed" in part (e). Plot the magnitude of the overall frequency response of the cascade system for $-\pi < \hat{\omega} < \pi$, and print a copy of the plot for your lab report.[3] Explain briefly why the frequency response magnitude has a notch, and explain why the gain (i.e., $|H(e^{j\hat{\omega}})|$) at $\hat{\omega} = 0$ and $\hat{\omega} = \pi$ is the same.

7   Filter Coefficients from Roots (Optional)

Compute the filter coefficients for the denominator $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ and numerator $B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}$ when the poles are:

$$p_1 = 0.75e^{j\pi/4}, \quad p_2 = 0.75e^{-j\pi/4},$$

and the zeros are:

$$z_1 = 1, \quad z_2 = -1$$

Use the following relationship:

$$H(z) = \frac{B(z)}{A(z)} = G\frac{(1 - z_1 z^{-1})(1 - z_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})} \tag{1}$$

where z_1 and z_2 are the zeros, and p_1 and p_2 are the poles defined above. Determine G so that the maximum magnitude of $H(e^{j\omega})$ is one. (Remember that MATLAB can multiply polynomials via its `conv` function.)

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[3]PeZ uses the normalized frequency scale: $-\frac{1}{2} < \hat{\omega}/(2\pi) < \frac{1}{2}$, which is the same as $-\pi < \hat{\omega} < \pi$.

Lab: PeZ - The z, n, and $\hat{\omega}$ Domains
WORKSHEET

Turn this page in at the end of your lab period.

Name: _____   Date of Lab: _____

Warm-up (Part 2): Implemented first-order system with `PeZ`:

Verified:_____   Date/Time:_____

Part	Observations
4(a)	$h[n]$ decays exponentially with no oscillations, $H(e^{j\hat{\omega}})$ has a hump at $\hat{\omega} = 0$
4(b)	
4(c)	
4(d)	
4(e)	
4(f)	
5(a)	
5(b)	
5(c)	
5(d)	

Part 6(a,b)	a_k	b_k
Filter 1		
Filter 2		
Cascade of 1 and 2		

Explanation of cascade frequency response:

Part 7. $G =$ [   ] Value of G so that the maximum magnitude of $H(e^{j\omega})$ is one.