The lab report/verification will be done by filling in the last page of this handout which addresses a list of observations to be made when using the PeZ GUI.

1 Introduction & Objective

The objective for this lab is to build an intuitive understanding of the relationship between the location of poles and zeros in the *z*-domain, the impulse response *h[n]* in the *n*-domain, and the frequency response $H(e^{j\hat{\omega}})$ (the *ω*-domain). A graphical user interface (GUI) called PeZ was written in LabVIEW for doing interactive explorations of the three domains[1]. PeZ is based on the system function, represented as a ratio of polynomials in z^{-1}, which can be expressed in either factored or expanded form as:

$$H(z) = \frac{B(z)}{A(z)} = G \frac{\prod_{k=1}^{M}\left(1 - z_k z^{-1}\right)}{\prod_{k=1}^{N}\left(1 - p_k z^{-1}\right)} = \frac{\sum_{k=1}^{M} b_k z^{-1}}{1 - \sum_{k=1}^{N} a_k z^{-1}}$$

2 Warm-up

If you have already installed the **dspfirstLV** toolbox[2], you run PeZ in LabVIEW by selecting **Help»Find Examples…** and clicking on the **Search** tab and entering **DSP**. Double-click on **DSP First** and then click on **Pez**. A control panel with a few buttons and a plot of the unit circle in the complex *z*-plane will pop up. See Figure 1.
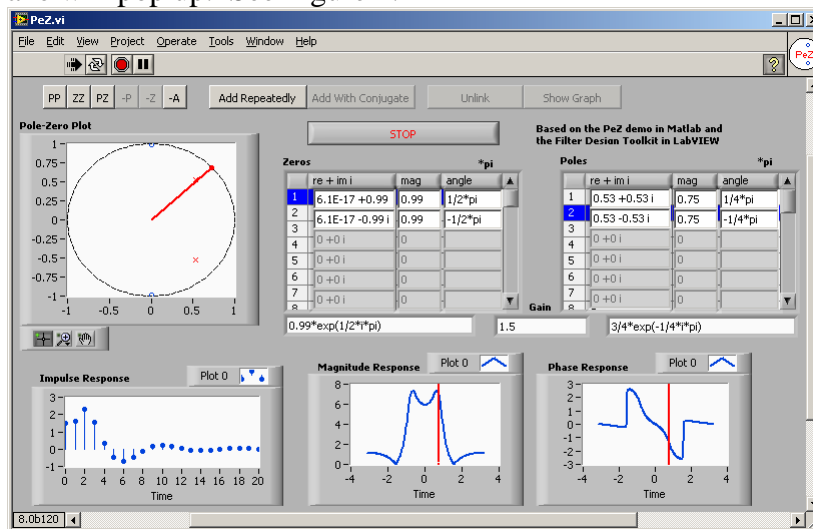

Figure 1 The PeZ GUI. This is for version 1.0. Other versions may look different.

Click the **Run** button and try the following. You can selectively place poles and zeros in the *z*-plane, and observe how their placement affects the impulse and frequency responses. Click the

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[1] The MATLAB version of PeZ was written by Craig Ulmer. The LabVIEW version was written by Josiah A. Yoder.
[2] If you haven't installed the toolbox, go to the CD-ROM and select **Getting Started**. Follow the instructions there.

PP button to add pole pairs. The **ZZ** button adds zero pairs, and the **PZ** button added pole/zero quads (try it). The other three buttons with –'s removes poles/zeros. (Try it).

PeZ is in a mode such that an individual pole/zero (pair) can be moved around and the corresponding $H\left(e^{j\hat{\omega}}\right)$ and *h[n]* plots will be updated as you drag the pole (or zero).

Since exact placement of poles and zeros with the mouse is difficult, you can type the pole (or zero) location in the box below the list of poles (or zeros). You can enter most any Matlab express, so entering a zero on the unit circle is as easy as entering `1*exp(j*2/3*pi)` in the box. Just be sure you also add the conjugate.

Removal of individual poles (or zeros) can also be performed by clicking on the pole (or zero) location in the list and clicking **Delete**. Note that all poles and/or zeros can be easily cleared by clicking on the **–A** button.

Play around with PeZ for a few minutes to gain some familiarity with the interface. Implement the following first-order system:

$$H(z) = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$

by placing its poles and zeros at the correct location in the *z*-plane. First try placing the pole and zero with the mouse, and then use the box entry feature.

What does the red line represent on the *z*-plane plot and the Magnitude and Phase plots? Try rotating it on the *z*-plane plot.

Instructor Verification (separate page)

3 Relationships between *z*, *n*, and ω domains

Work through the following exercises and keep track of your observations by filling in the worksheet at the end of this assignment. In general, you want to make note of the following quantities:

a)   How does *h[n]* change with respect to its rate of decay? For example, when $h[n] = a^n u[n]$, the impulse response will fall off more rapidly when *a* is smaller.

b)   If *h[n]* exhibits an oscillation period, what is the period? Also, estimate the decay rate of the "envelope."

c)   How does $H\left(e^{j\hat{\omega}}\right)$ change with respect to peak location and peak width?

Note: review the "Three-Domains – FIR" under the Demos link for chapter 7 and "Three-Domains – IIR" under the Demos link for chapter 8 for examples of these relationships.

4 Lab Exercise: Real Poles

a)   Use PeZ to place a single pole at $z = \frac{1}{2}$. You may have to use the edit box to get the location exactly right. Use the plots for this case as the reference for answering the next four parts.

b)   Move the pole close to the origin (still on the real axis). You can do this by clicking on the pole and dragging it to the new location. Describe the changes in the impulse response *h[n]* and the frequency response $H\left(e^{j\hat{\omega}}\right)$.

c)   The impulse response and frequency response plots are updated while you move a pole (or zero). Click on the pole you want to move and start to drag it slowly. Watch for the

update of the plots in the secondary window. Move the real pole slowly from to $z = 1$ and observe the changes in the impulse response *h[n]* and the frequency response $H\left(e^{j\hat\omega}\right)$.

d) Place the pole exactly on the unit circle (or maybe just inside at a radius of 0.99999999). Describe the changes in *h[n]* and $H\left(e^{j\hat\omega}\right)$. What do you expect to see for $H\left(e^{j\hat\omega}\right)$?

e) Move the pole outside the unit circle. Describe the changes in *h[n]*. Explain how the appearance of *h[n]* validates the statement that the system is not stable. In this case, the frequency response $H\left(e^{j\hat\omega}\right)$ is not legitimate because the system is no longer stable.

f) In general, where should poles be placed to guarantee system stability? By stability we mean that the system's output does not blow up.

5 Lab Exercise: Complex Poles and Zeros

PeZ assumes real coefficients for the numerator and denominator polynomials. Therefore, if we enter a complex pole or zero, PeZ will automatically insert second root at the conjugate location. For example, if we place a root at $z = \frac{1}{3} + j\frac{1}{2}$, then we will also get one at $z = \frac{1}{3} - j\frac{1}{2}$.

a) What property of the polynomial coefficients of $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ will guarantee that the roots come in conjugate pairs?

b) Clear all the poles and zeros from PeZ. Now place a pole with magnitude 0.85 at an angle of 45°; and then two zeros at the origin. Note that PeZ automatically places a conjugate pole in the *z*-domain. The frequency response has a peak -- record the frequency (location) of this peak. Hint: Adjust the red ray on the *z*-plane.

c) Change the angle of the pole: move the pole to 90°, then 135 °. Describe the changes in $H\left(e^{j\hat\omega}\right)$. Concentrate on the location of the peak.

Next, we will put complex zeros on the unit circle to see the effect on $H\left(e^{j\hat\omega}\right)$.

d) Clear all poles and zeros from PeZ. Now place zeros at the following locations: $z_1 = -1$, $z_2 = 0 - j$ and $z_3 = 0 + j$ (remember that conjugate pairs such as z_2 and z_3 will be entered simultaneously). Judging from the impulse and frequency responses what type of filter have you just implemented?

6 Lab Exercise: Filter Design

In this section, we will use PeZ to place the poles and zeros of *Hz* to make a filter with a desirable frequency response. *Filter design* is a process that selects the coefficients a_k and b_k to accomplish a given task. The task here is to create a filter that has a very narrow "notch." This filter would be useful for removing one frequency component while leaving others undisturbed. The *notch filter* can be synthesized from the cascade of two simpler filters shown in Figure 2.

(a) FIR Filter

(b) Feedback Filter
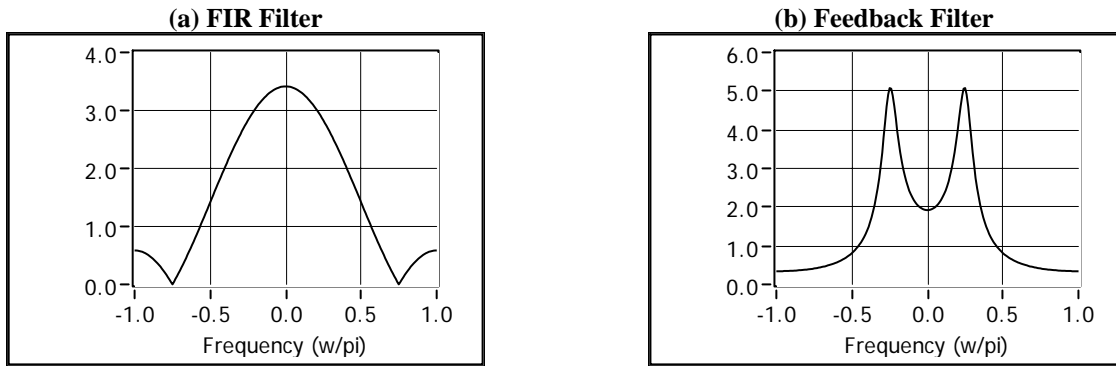
Figure 2 Magnitude response of two unknown filters. Frequency axis is normalized $\hat{\omega}/\pi$. Use PeZ to help you find the filter coefficients that will match these frequency responses as closely as possible. (a) Second-order FIR filter. (b) Second-order IIR filter.

a) Start the process by using PeZ to design each of the filters given in Figure 2. (You will have to determine the locations of the poles and zeros from the plots in Figure 2.) Both filters are second-order. Make sure that you enter the poles and zeros precisely. PeZ will do the conversion between root locations and polynomial coefficients. You can check your results by also calculating the filter coefficients by hand (see the next section on polynomials with complex coefficients). Record the coefficients of your filters in the table provided.

Note: Use PeZ or `freqz` to verify that the frequency response of each filter is correct.

b) Now use PeZ to connect the filters together in a cascade. Place the poles and zeros, and then view the frequency response. Determine the filter coefficients for the overall cascaded filter *Hz*.

c) Use `freqz()` to determine the frequency response of the cascade of the two filters that you "designed" in part (e). Plot the magnitude of the overall frequency response of the cascade system for $-\pi < \hat{\omega} < \pi$, and print a copy of the plot for your lab report. Explain briefly why the frequency response magnitude has a notch, and explain why the gain (i.e., $\left|H\left(e^{j\hat{\omega}}\right)\right|$ at $\hat{\omega}=0$ and $\hat{\omega}=\pi$ is the same.

7   Filter Coefficients from Roots (Optional)

Compute the filter coefficients for the denominator $A(z) = 1 - a_1 z^{-1} - a_2 z^{-2}$ and numerator $B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}$ when the poles are:

$$p_1 = 0.75e^{j\pi/4}, \quad p_2 = 0.75e^{-j\pi/4},$$

and the zeros are:

$$z_1 = 1, \quad z_2 = -1$$

Use the following relationship:

$$H(z) = \frac{B(z)}{A(z)} = G\frac{\left(1 - z_1 z^{-1}\right)\left(1 - z_2 z^{-1}\right)}{\left(1 - p_1 z^{-1}\right)\left(1 - p_2 z^{-1}\right)}$$

where z_1 and z_2 are the zeros, and p_1 and p_2 are the poles defined above. Determine G so that the maximum magnitude of $H\left(e^{j\hat{\omega}}\right)$ is one. (Remember that LabVIEW can multiply polynomials via its **Multiply Polynomicals** function.)

Lab 11
WORKSHEET

Turn this page in at the end of your lab period.

Name: _____ Date of Lab: _____

Warm-up (Part 2): Implemented first-order system with PeZ:

   Verified: _____ Date/Time: _____

Part	Observations
4(a)	$h[n]$ decays exponentially with no oscillations, $H\left(e^{j\hat{\omega}}\right)$ has a hump at $\hat{\omega}=0$.
4(b)	
4(c)	
4(d)	
4(e)	
4(f)	
5(a)	
5(b)	
5(c)	
5(d)	

Part 6 (a,b)	a_k	b_k
Filter 1		
Filter 2		
Cascade of 1 and 2		

Explanation of cascade frequency response: