Signal Processing First

Lecture 24
Amplitude Modulation (AM)

LECTURE OBJECTIVES

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM
- Reading: Chapter 12, Section 12-2

Table of Easy FT Properties

Linearity Property

$$ax_1(t) + bx_2(t) \Leftrightarrow aX_1(j\omega) + bX_2(j\omega)$$

Delay Property

$$x(t - t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j(\frac{\omega}{a}))$$

Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Frequency Shifting Property

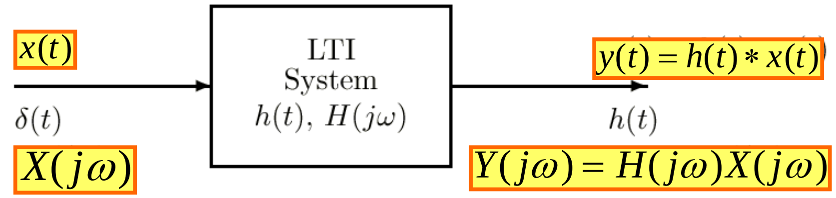
$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

$$\int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt$$

$$= X(j(\omega-\omega_0))$$

$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & elsewhere \end{cases}$$

Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

corresponds to **MULTIPLICATION** in the frequency-domain $Y(j\omega) = H(j\omega)X(j\omega)$

Cosine Input to LTI System

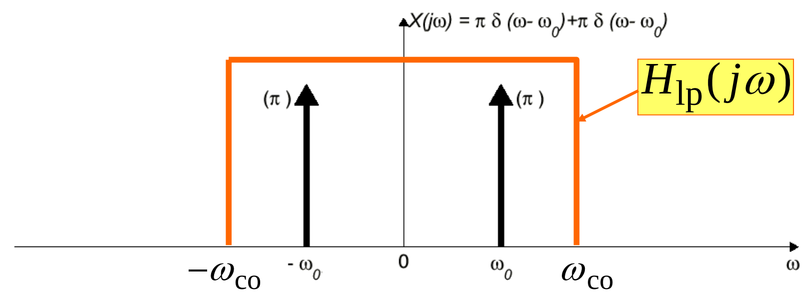
$$Y(j\omega) = H(j\omega)X(j\omega)$$

$$= H(j\omega)[\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$$

$$= H(j\omega_0)\pi\delta(\omega-\omega_0) + H(-j\omega_0)\pi\delta(\omega+\omega_0)$$

$$\begin{aligned} y(t) &= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t} \\ &= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t} \\ &= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0)) \end{aligned}$$

Ideal Lowpass Filter

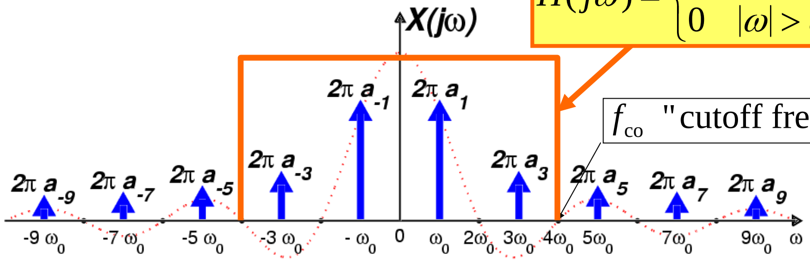


$$y(t) = x(t) \quad \text{if } \omega_0 < \omega_{co}$$

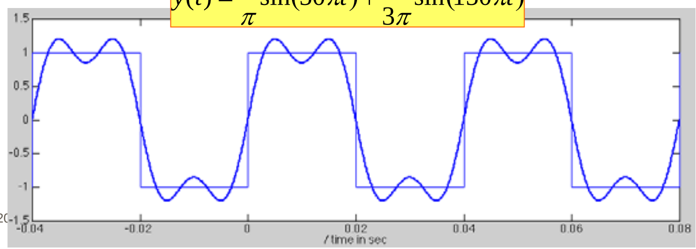
$$y(t) = 0 \quad \text{if } \omega_0 > \omega_{co}$$

Ideal LPF: Fourier Series

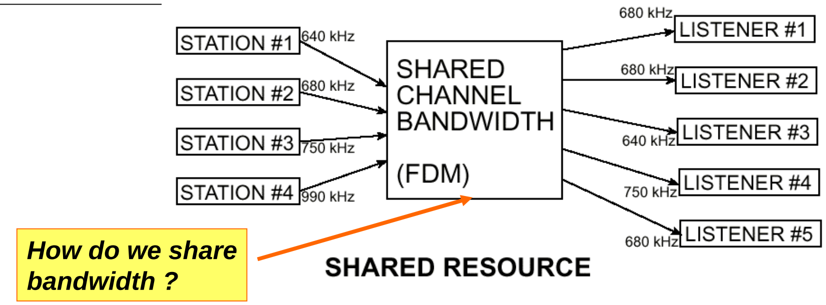
$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$



$$y(t) = \frac{4}{\pi}\sin(50\pi t) + \frac{4}{3\pi}\sin(150\pi t)$$



The way communication systems work



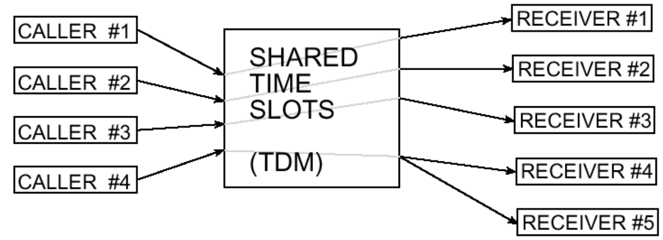


Table of FT Properties

$$x(t) * h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

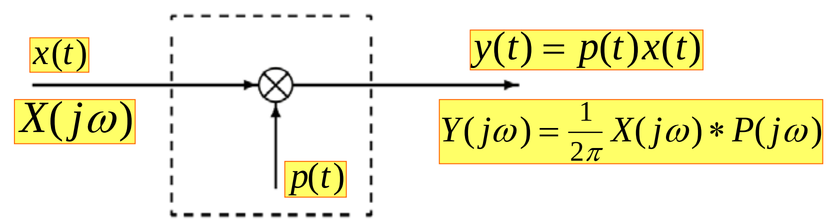
$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)P(j(\omega - \theta))d\theta$$

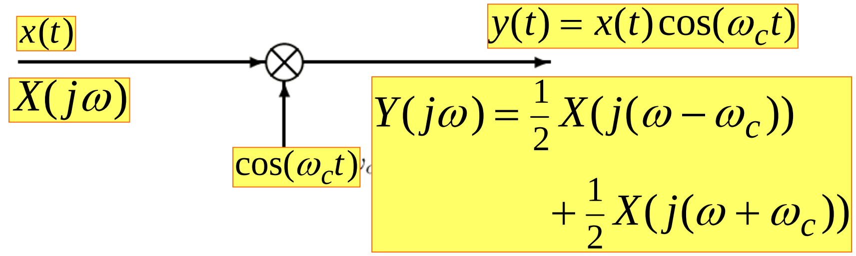
$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \tfrac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$p(t) = \cos(\omega_c t) \Leftrightarrow$$
$$P(j\omega) = \pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2\pi} X(j\omega) * \left[\pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)\right]$$

$$Y(j\omega) = \tfrac{1}{2} X(j(\omega - \omega_c)) + \tfrac{1}{2} X(j(\omega + \omega_c))$$

Amplitude Modulator



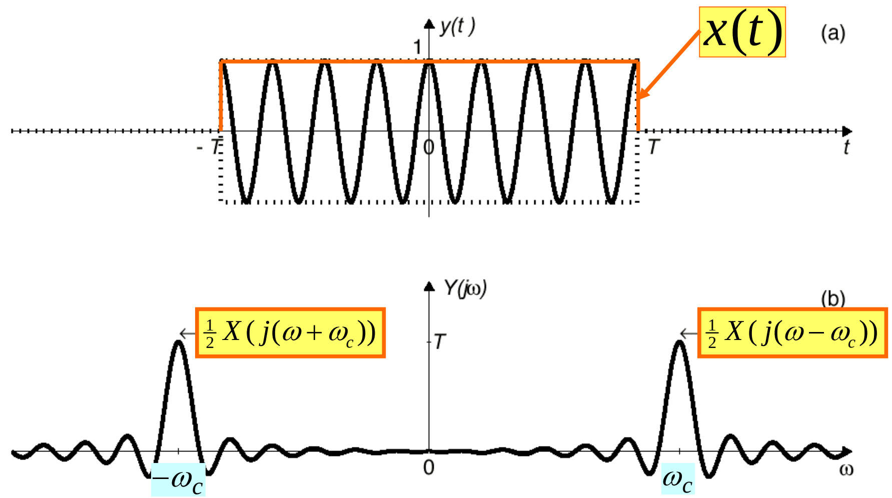
- *x(t)* modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of *X(jω).*

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2} X(j(\omega - \omega_c)) + \tfrac{1}{2} X(j(\omega + \omega_c))$$

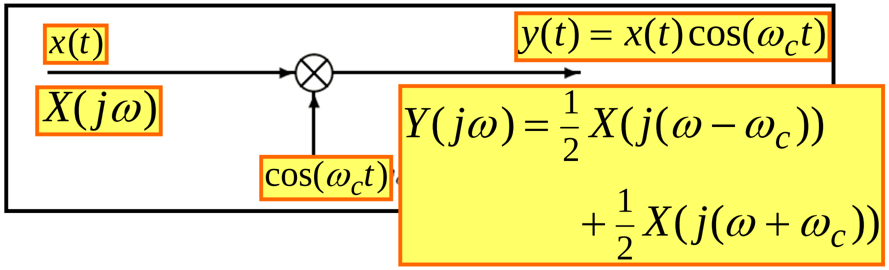
$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \quad \Leftrightarrow \quad X(j\omega) = 2\frac{\sin(\omega T)}{(\omega)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \frac{\sin((\omega - \omega_c)T)}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c)T)}{(\omega + \omega_c)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2} X(j(\omega - \omega_c)) + \tfrac{1}{2} X(j(\omega + \omega_c))$$



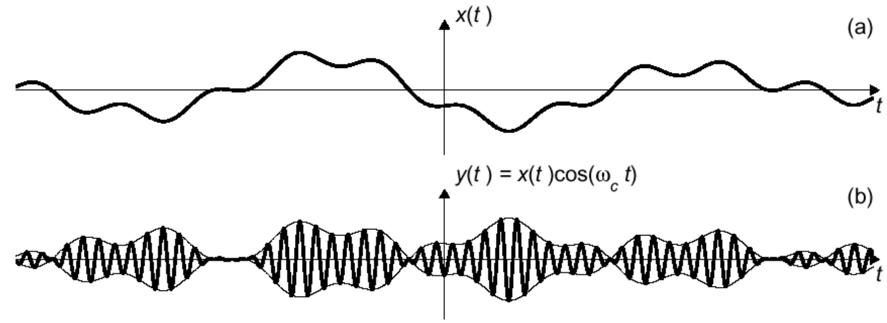
DSBAM Modulator



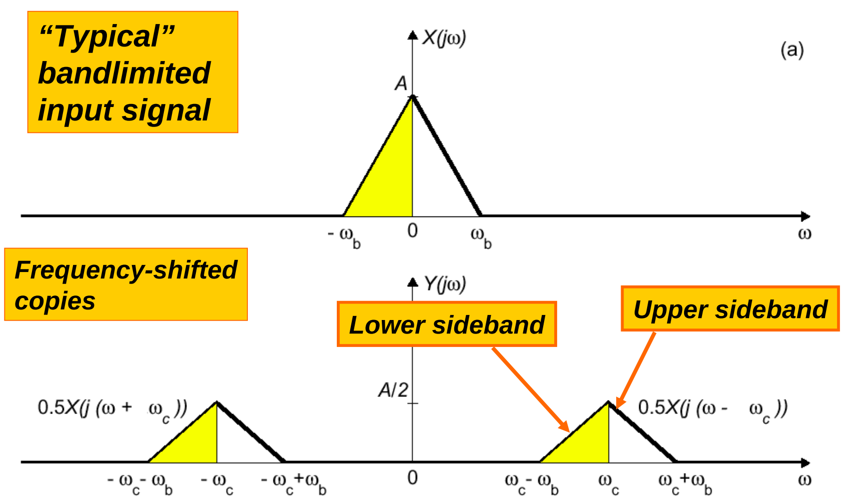
- If $X(j\omega)=0$ for $|\omega|>\omega_b$ and $\omega_c>\omega_b$,the result in the frequency-domain is two shifted and scaled **exact copies** of $X(j\omega)$.

DSBAM Waveform

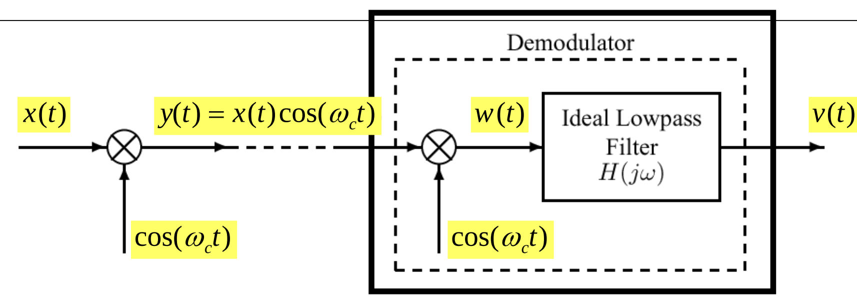
- In the time-domain, the "envelope" of sine-wave peaks follows $|x(t)|$



Double Sideband AM (DSBAM)



DSBAM DEmodulator

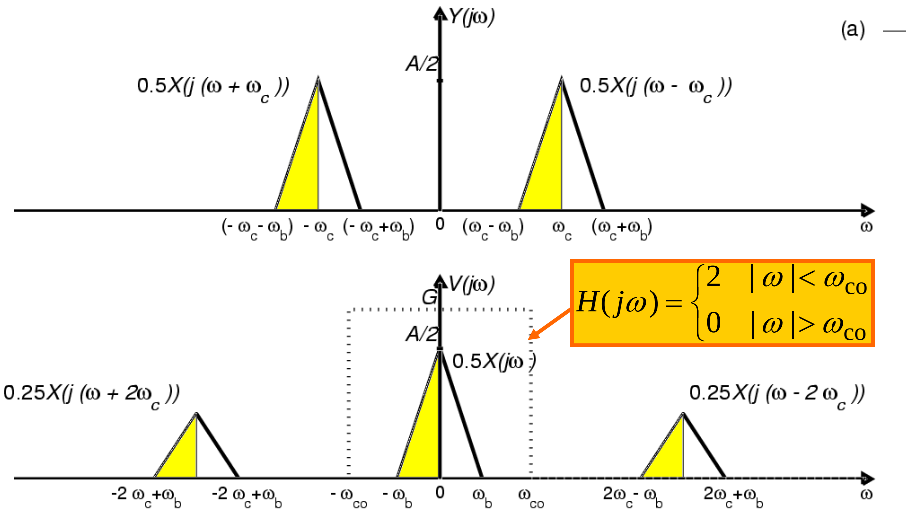


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega-2\omega_c)) + \frac{1}{4}X(j(\omega+2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

DSBAM Demodulation



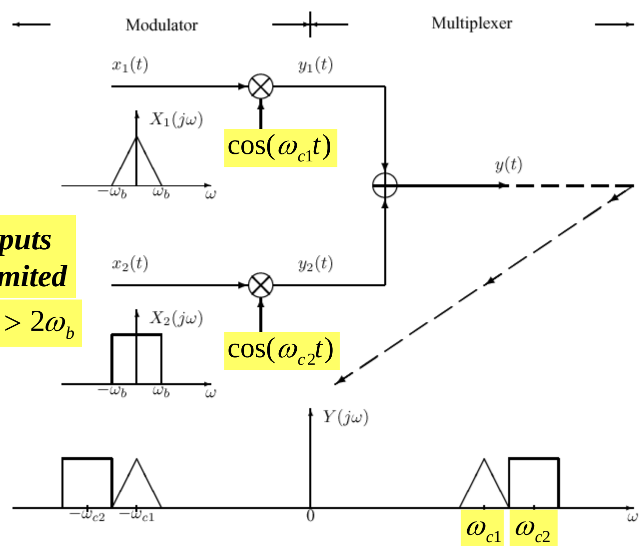
$$H(j\omega)=\begin{cases}2 & |\omega|<\omega_{co}\\ 0 & |\omega|>\omega_{co}\end{cases}$$

$$V(j\omega)=H(j\omega)W(j\omega)=X(j\omega)\ \text{ if }\ \omega_b<\omega_{co}<2\omega_c-\omega_b$$

Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple bandlimited signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

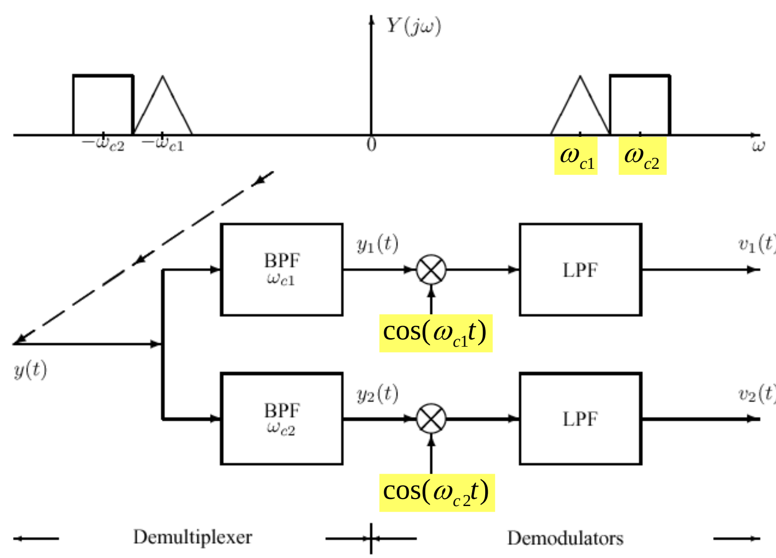
FDM Block Diagram (Xmitter)



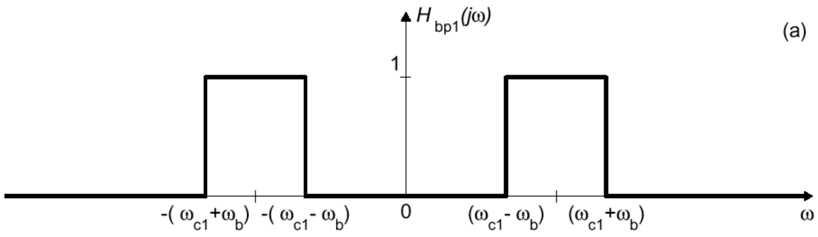
Spectrum of inputs must be bandlimited

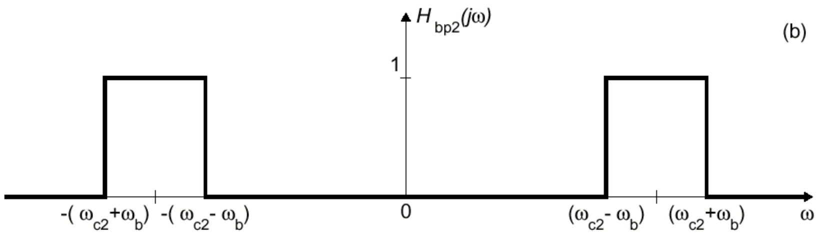
Need $|\omega_{c2}-\omega_{c1}|>2\omega_b$

Frequency-Division De-Mux

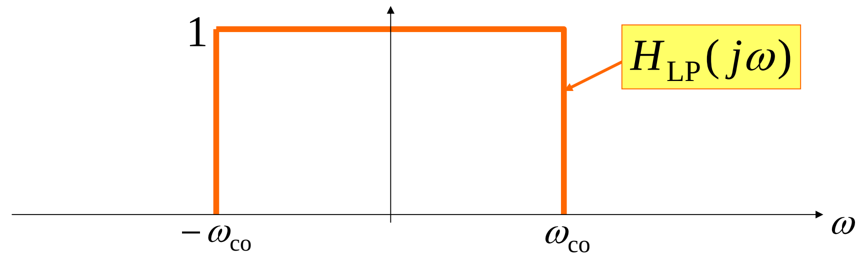


Bandpass Filters for De-Mux





Pop Quiz: FT thru LPF



Input $x(t) \quad \leftrightarrow \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 4\pi\delta(\omega - 30\pi k)$

If the output is $y(t) = 2$, then find a value for ω_{co}