Example 10-16: Consider the following system

$$y[n] = y[n-1] - y[n-2] + 2x[n] + 2x[n-1]$$

whose system function is

$$H(z) = \frac{2 + 2z^{-1}}{1 - z^{-1} + z^{-2}} = \frac{2(1 + z^{-1})}{(1 - e^{j\pi/3}z^{-1})(1 - e^{-j\pi/3}z^{-1})} \tag{10.13}$$

A pole-zero plot for this $H(z)$ was already given in Fig. **??**. Using the partial fraction expansion technique, we can write $H(z)$ in the form

$$H(z) = \frac{\left(\dfrac{2 + 2e^{-j\pi/3}}{1 - e^{-j2\pi/3}}\right)}{1 - e^{j\pi/3}z^{-1}} + \frac{\left(\dfrac{2 + 2e^{j\pi/3}}{1 - e^{j2\pi/3}}\right)}{1 - e^{-j\pi/3}z^{-1}} = \frac{2e^{-j\pi/3}}{1 - e^{j\pi/3}z^{-1}} + \frac{2e^{j\pi/3}}{1 - e^{-j\pi/3}z^{-1}}$$

The right-hand side is the sum of two first-order terms for which entry 6 in Table **??** applies with $a = e^{j\pi/3}$ and $a = e^{-j\pi/3}$. The individual terms are complex conjugates, so their sum is purely real:

$$h[n] = 2e^{-j\pi/3}e^{j(\pi/3)n}u[n] + 2e^{j\pi/3}e^{-j(\pi/3)n}u[n] = 4\cos\left((2\pi/6)(n-1)\right)u[n]$$

The two complex exponentials with frequencies $\pm\pi/3$ combine to form the cosine. The impulse response is plotted in Fig. **??**.

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