Example 6-13: To illustrate the effect of noninteger delay with the running-average filter, consider the cosine signal $x[n] = \cos(0.2\pi n)$, which could have resulted in Fig **??** from sampling the signal $x(t) = \cos(200\pi t)$ with sampling rate $f_s = 1000$ Hz. Figure **??**(a) shows $x(t)$ and $x[n]$. If $x[n]$ is the input to a 5-point running-average filter, the steady-state part of the output is

$$y_5[n] = \frac{\sin(0.2\pi(5/2))}{5\sin(0.2\pi/2)} \cos(0.2\pi n - 0.2\pi(2)) = 0.6472\cos(0.2\pi(n-2))$$

For this filter output, the output of the *ideal* D-to-C converter in Fig. **??**(b) would be

$$y_5(t) = y_5[n]\big|_{n=1000t} = 0.6472\cos(200\pi(t-0.002))$$

The delay is 2 samples. On the other hand, if the same signal $x[n]$ is the input to a 4-point running-average system, the steady-state part of the output (Fig. **??**(c)) is

$$y_4[n] = \frac{\sin(0.2\pi(4/2))}{4\sin(0.2\pi/2)} \cos(0.2\pi n - 0.2\pi(^3\!/_2)) = 0.7694\cos(0.2\pi(n - ^3\!/_2))$$

Now the delay is $^3\!/_2$ samples, so we cannot write $y_4[n]$ as an integer shift with respect to the input sequence. In this case, the "$^3\!/_2$ samples" delay introduced by the filter can be interpreted in terms of the corresponding output of the *ideal* D-to-C converter in Fig. **??**(c), which in this case would be

$$y_4(t) = y_4[n]\big|_{n=1000t} = 0.7694\cos(200\pi(t-0.0015))$$

Figure **??** shows the input and the corresponding outputs $y_5[n]$ and $y_5(t)$ and $y_4[n]$ and $y_4(t)$. In all cases, the solid gray curve is the continuous-time cosine signal that would be reconstructed by the ideal D-to-C converter for the given discrete-time signal.

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