DSP First 2/e

Lecture 5A:
Operations on the Spectrum

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, Section 3-3 (DSP-First 2/e)
- Other Reading:
  - Appendix A: Complex Numbers

LECTURE OBJECTIVES

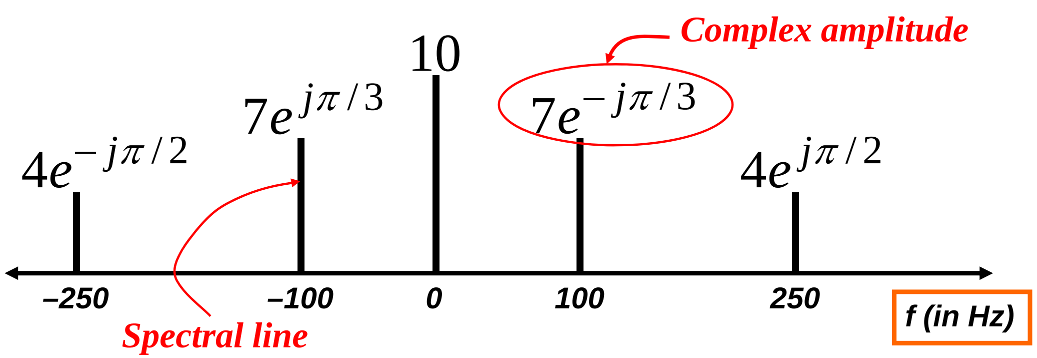
- **Operations** on a time-domain signal x(t) have a **SIMPLE form** in the frequency-domain
- SPECTRUM Representation has lines at:

  (A_k, φ_k, f_k)
- Represents Sinusoid with DIFFERENT Frequencies

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

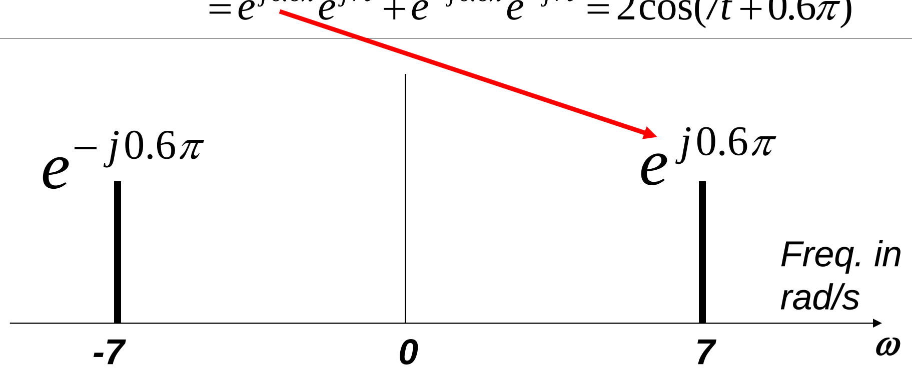
Recall FREQUENCY DIAGRAM

- Used to visualize relationship between frequencies, amplitudes and phases
- Plot Complex Amplitude vs. Freq



GRAPHICAL SPECTRUM

$$-2\sin(7t+0.1\pi) = \tfrac{1}{2}2e^{j\pi}e^{-j0.5\pi}e^{j0.1\pi}e^{j7t} + \tfrac{1}{2}2e^{-j\pi}e^{j0.5\pi}e^{-j0.1\pi}e^{-j7t}$$
$$= e^{j0.6\pi}e^{j7t} + e^{-j0.6\pi}e^{-j7t} = 2\cos(7t+0.6\pi)$$



AMPLITUDE, PHASE & FREQUENCY are shown

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General Spectrum

- 2M + 1 spectrum components:

$$x(t) = \sum_{k=-M}^{M} a_k e^{j2\pi f_k t}$$

- At $f = f_k$ the complex amplitude is a_k
  - usually, for real x(t) $f_0 = 0$

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OPERATIONS on SPECTRUM

- Adding DC, or amplitude scaling
- Adding two (or more) signals
- Time-Shifting
  - Multiply in frequency by complex exponential
- Differentiation of x(t)
  - Multiply in frequency-domain by (jω)
- Frequency Shifting
  - Multiply in time-domain by sinusoid

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Scaling or Adding a constant

- Adding DC

$$x(t) + c = \sum_{k\neq 0} a_k e^{j2\pi f_k t} + \underbrace{a_0 e^{j2\pi(0)t} + c e^{j2\pi(0)t}}_{\text{new DC is } a_0 + c}$$

- Scaling

$$\gamma x(t) = \gamma \sum_{k=-M}^{M} a_k e^{j2\pi f_k t} = \sum_{k=-M}^{M} (\gamma a_k) e^{j2\pi f_k t}$$

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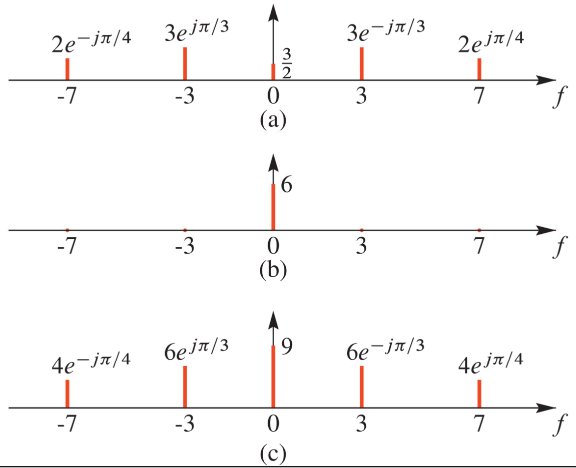
Scaling and Adding a constant

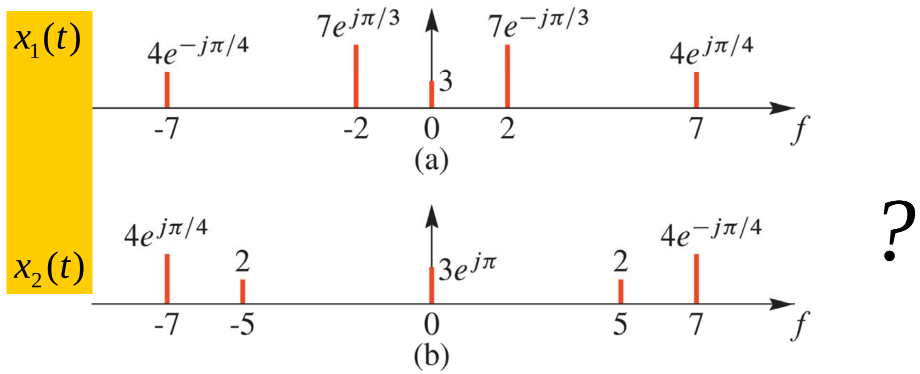
$$2x(t)+6=\sum_{k\neq 0} 2a_k e^{j2\pi f_k t} + \underbrace{2a_0+6}_{\text{new DC}}$$



Adding Two Signals (1)

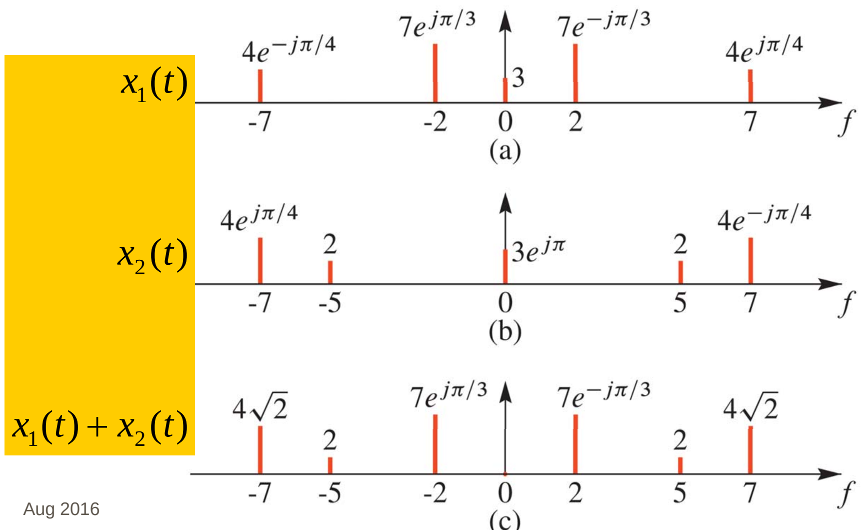
- Adding signals with same fundamental

$$x_1(t)+x_2(t)=\sum_{k=-M}^{M} a_{1k}e^{j2\pi f_k t}+\sum_{k=-M}^{M} a_{2k}e^{j2\pi f_k t}=\sum_{k=-M}^{M}(a_{1k}+a_{2k})e^{j2\pi f_k t}$$



Adding Two Signals (2)

- Adding signals with same fundamental



Time Shifting x(t)

- Time Shifting

$$x(t-\tau_d)=\sum_{k=-M}^{M} a_k e^{j2\pi f_k (t-\tau_d)} = \sum_{k=-M}^{M} \underbrace{(a_k e^{-j2\pi f_k \tau_d})}_{b_k} e^{j2\pi f_k t}$$

$$y(t)=\sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$

- Multiply Spectrum complex amplitudes by a complex exponential

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Differentiating x(t)

- Take **derivative** of the Signal x(t)

$$\frac{d}{dt}x(t) = \sum_{k=-M}^{M} a_k (j2\pi f_k) e^{j2\pi f_k t} = \sum_{k=-M}^{M} \underbrace{(j2\pi f_k) a_k}_{b_k} e^{j2\pi f_k t}$$

$$y(t) = \sum_{k=-M}^{M} b_k e^{j2\pi f_k t}$$

- Multiply complex amplitudes by "jω"="j2πf"

Frequency Shifting x(t)

- Multiply x(t) by Complex Exponential
  ➔Frequency Shifting

$$y(t) = Ae^{j\varphi} e^{j2\pi f_c t} x(t)$$

$$y(t) = \sum_{k=-M}^{M} Ae^{j\varphi} e^{j2\pi f_c t} a_k e^{j2\pi f_k t}$$

$$= \sum_{k=-M}^{M} (a_k A e^{j\varphi}) e^{j2\pi (f_k + f_c) t}$$

- Spectrum components shifted: $f_k \rightarrow f_k + f_c$

Frequency Shifting x(t)