DSP First, 2/e

Lecture 11

FIR Filtering Intro

READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, Sects. 5-1, 5-2, 5-3 & 5-4 (partial)
- Other Reading:
  - Next Lecture: Ch. 5, Sects 5-4, 5-6, 5-7 & 5-8
    - CONVOLUTION

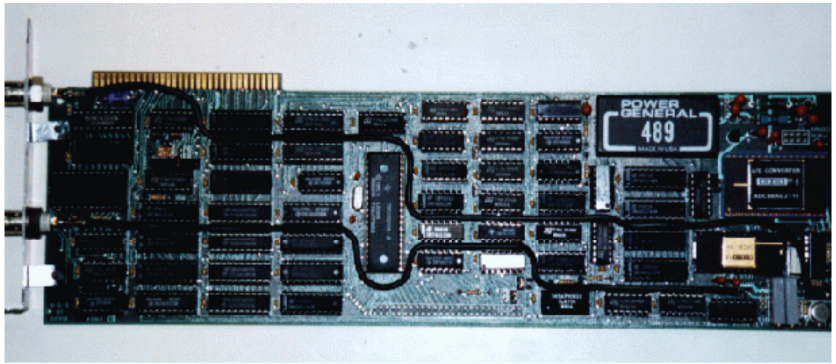
LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output y[n] from the input signal, x[n]

DIGITAL FILTERING

- Characterized SIGNALS (Fourier series)
- Converted to DIGITAL (sampling)
- Today: How to PROCESS them (DSP)?
- CONCENTRATE on the COMPUTER
  - ALGORITHMS, SOFTWARE (MATLAB) and HARDWARE (DSP chips, VLSI)

The TMS32010, 1983

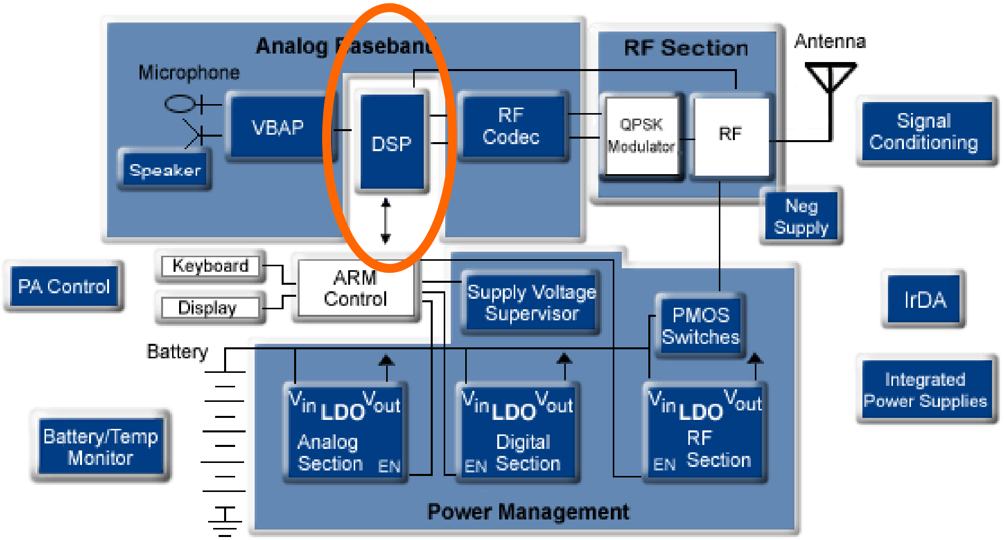


First PC plug-in board from Atlanta Signal Processors Inc.

Rockland Digital Filter, 1971

Cost was about the same as the price of a small house.

Digital Cell Phone (ca. 2000)



Now, digital cameras and video streaming rely on DSP algorithms

DISCRETE-TIME SYSTEM

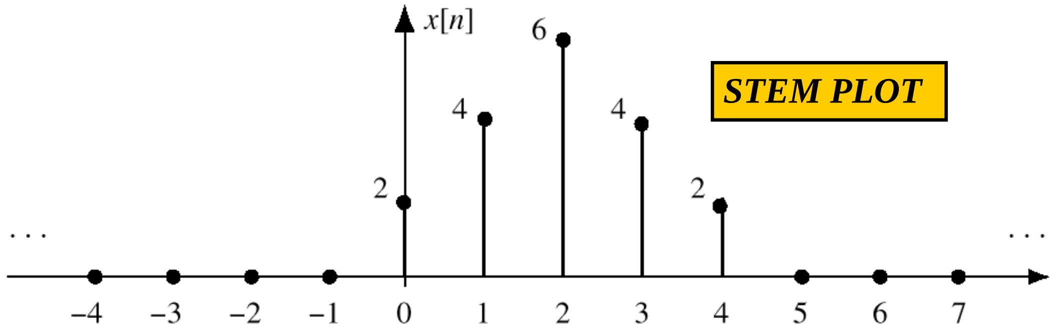
- OPERATE on x[n] to get y[n]
- WANT a **GENERAL** CLASS of SYSTEMS
  - **ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM

D-T SYSTEM EXAMPLES

- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"

DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by "n"



STEM PLOT

3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "n"

the following input–output equation

Make a TABLE

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

n=0 $y[0] = \tfrac{1}{3}(x[0] + x[1] + x[2])$

n=1 $y[1] = \tfrac{1}{3}(x[1] + x[2] + x[3])$

INPUT SIGNAL

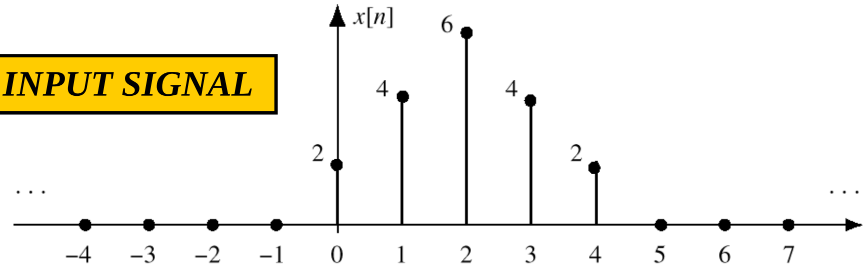


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

OUTPUT SIGNAL

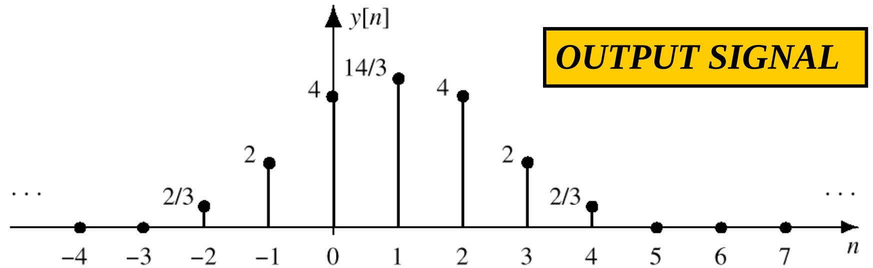


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

- SLIDE a WINDOW across x[n]

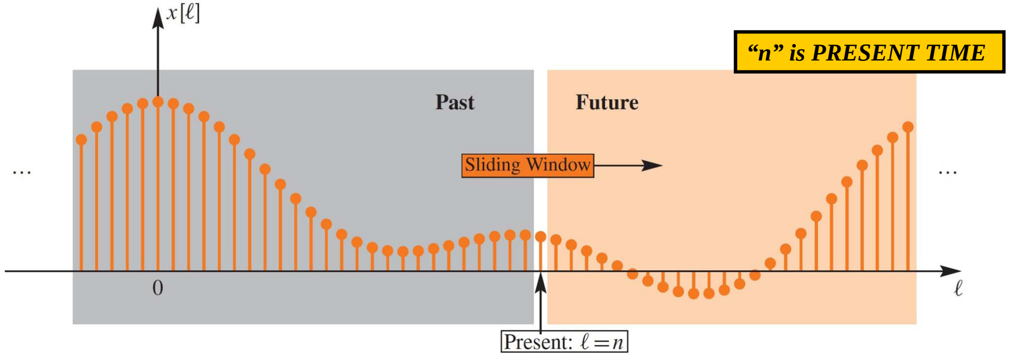


Figure 5-3 Filter calculation at the present time ($\ell = n$) uses values within a sliding window. Gray shading indicates the past ($\ell < n$); orange shading, the future ($\ell > n$). Here, the sliding window encompasses values from both the future and the past.

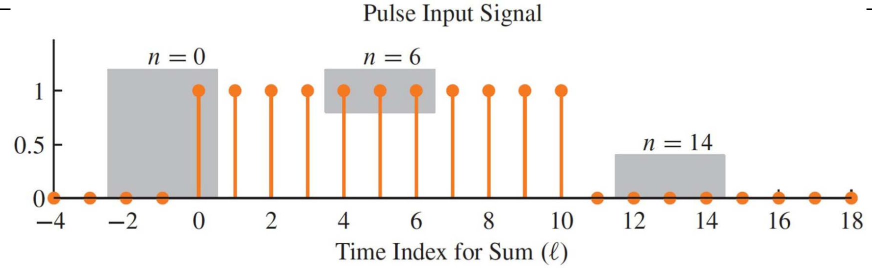
ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

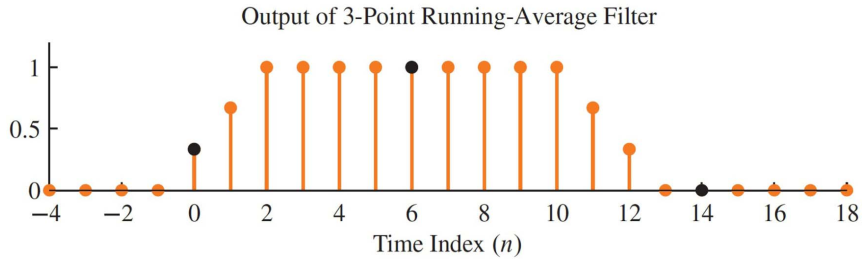
n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

CAUSAL 3-pt AVERAGER



(a)

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$



(b)

GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$
$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

GENERAL CAUSAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

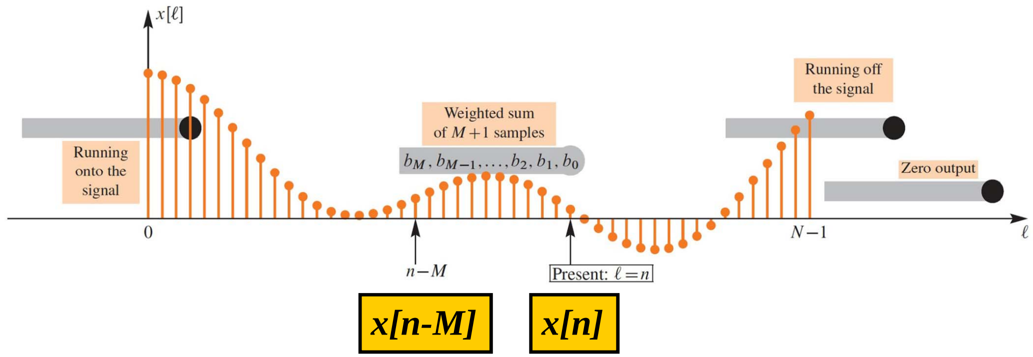
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is M
- FILTER **"LENGTH"** is L = M+1
  - NUMBER of FILTER COEFFS is L

GENERAL CAUSAL FIR FILTER

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$



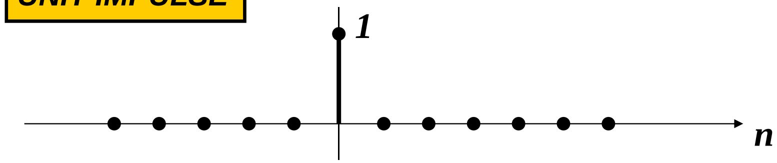
FILTERED STOCK SIGNAL

SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE (LATER)**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE SIGNAL δ[n]

n	...	−2	−1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

δ[n] is NON-ZERO When its argument is equal to ZERO

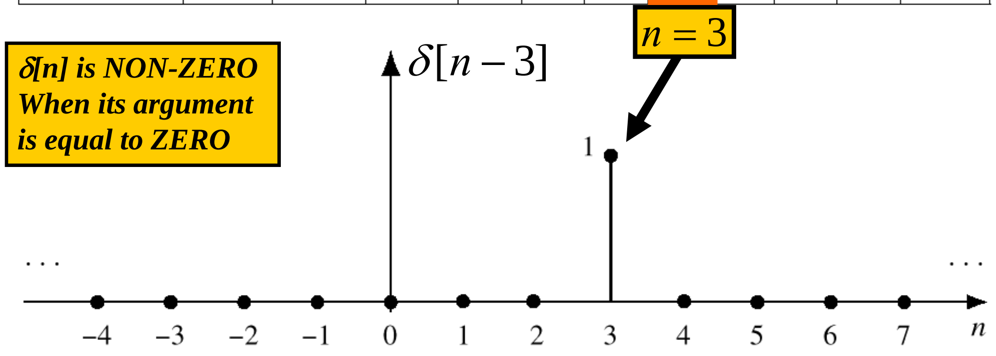
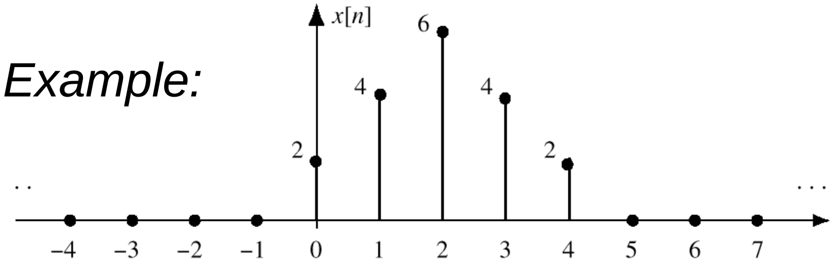


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

Sequence Representation

Example:



$$x[n=0]=x[0]=2 \qquad x[n=1]=x[1]=4$$

$$x[n=2]=x[2]=6 \qquad x[n=3]=x[3]=4$$

$$x[n]=\cdots+\ 0\ \delta[n+1]+\ 2\ \delta[n]+\ 4\ \delta[n-1]$$
$$+\ 6\ \delta[n-2]+4\ \delta[n-3]+\cdots$$

UNIT IMPULSE RESPONSE

- FIR filter description usually given in terms of coefficients b_k

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

- Can we describe the filter using a <u>**SIGNAL**</u> instead?
- What happens if input is a unit impulse?

Example: 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES

$$y[n]=\tfrac{1}{4}(x[n]+x[n-1]+x[n-2]+x[n-3])$$

- *INPUT = UNIT IMPULSE SIGNAL = δ[n]*

$$x[n]=\delta[n]$$
$$y[n]=\tfrac{1}{4}\delta[n]+\tfrac{1}{4}\delta[n-1]+\tfrac{1}{4}\delta[n-2]+\tfrac{1}{4}\delta[n-3]$$

- *OUTPUT is called **"IMPULSE RESPONSE"***
  - *Denoted h[n]=y[n] when x[n]=δ[n]*

Unit Impulse Response

$$y[n] = \tfrac{1}{4}x[n] + \tfrac{1}{4}x[n-1] + \tfrac{1}{4}x[n-2] + \tfrac{1}{4}x[n-3]$$

n	-3	-2	-1	0	1	2	3	4	5
$x[n]$	0	0	0	1	0	0	0	0	0
$y[n]$	0	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

$$x[n] = \delta[n]$$

$$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$

$$= h[n]$$

SUM of Shifted Impulses

$$h[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3] + 0\delta[n-4]$$

n	-1	0	1	2	3	4	5	6	7
$h[n]$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	0
$h[0]\delta[n]$	0	$\frac{1}{4}$	0	0	0	0	0	0	0
$h[1]\delta[n-1]$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
$h[2]\delta[n-2]$	0	0	0	$\frac{1}{4}$	0	0	0	0	0
$h[3]\delta[n-3]$	0	0	0	0	$\frac{1}{4}$	0	0	0	0
$h[4]\delta[n-4]$	0	0	0	0	0	0	0	0	0

FIR IMPULSE RESPONSE

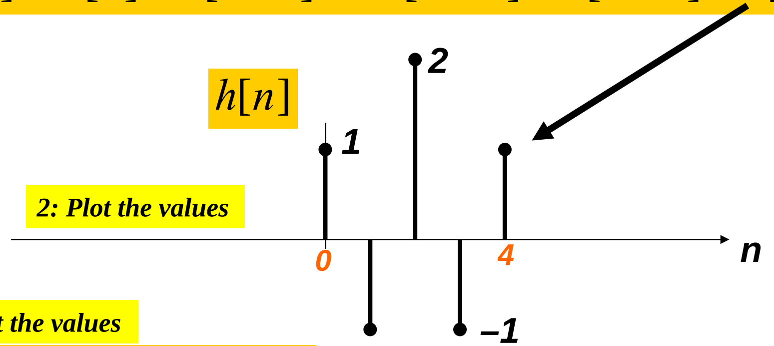
$$y[n] = \sum_{k=0}^{M} b_k x[n-k] \qquad h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

n	$n<0$	0	1	2	3	$\dots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\dots$	b_M	0	0

3 Ways to Represent the FIR filter

1 Use SHIFTED IMPULSES to write ***h*[*n*]**

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



2: Plot the values

3: List the values

$$b_k = \{1, -1, 2, -1, 1\}$$

True for any signal, x[n]

FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

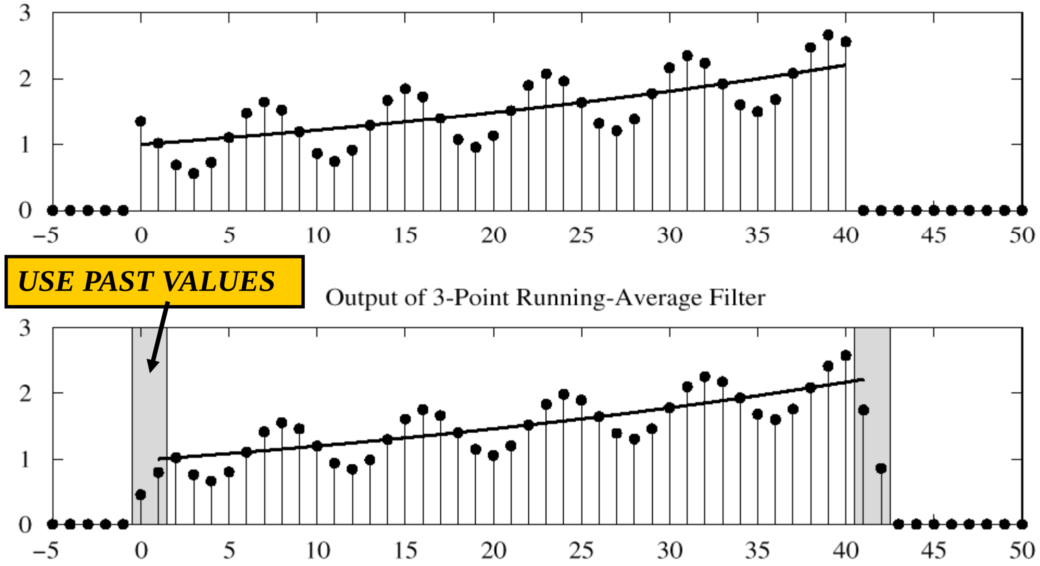
$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- *3-point AVERAGER*
  - *Changes A slightly*

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$



7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$