DSP First, 2/e

Lecture 16a
FIR Filter Design via Windowing

READING ASSIGNMENTS

- This Lecture:
  - Chapter 7, Sects 7-3 and 7-4

Lecture Objectives

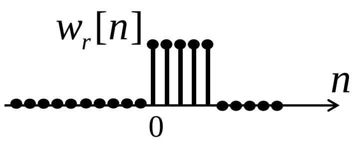
- Approximate ideal filters
- Introduce the concept of windowing
- Truncate ideal h[n] with a window
- Filter specs: ripples & band edges

Windows

- Finite-Length signal (L) with positive values
  - Extractor
  - Truncator

Rectangular Window

$$w_r[n] = \begin{cases} 0 & n < 0 \\ 1 & 0 \le n < L \\ 0 & n \ge L \end{cases}$$



$$w_r[n]x[n+n_0] = \begin{cases} 0 & n < 0 \\ x[n+n_0] & 0 \le n < L \\ 0 & n \ge L \end{cases}$$

Window Truncates Ideal h[n]

- sinc is inverse DTFT of ideal LPF

$$h[n] = \frac{\sin(\hat{\omega}_b n)}{\pi n} \qquad -\infty < n < \infty$$

  - Truncate: Multiply sinc by a window
  - Finite h[n] has length (L) = window length

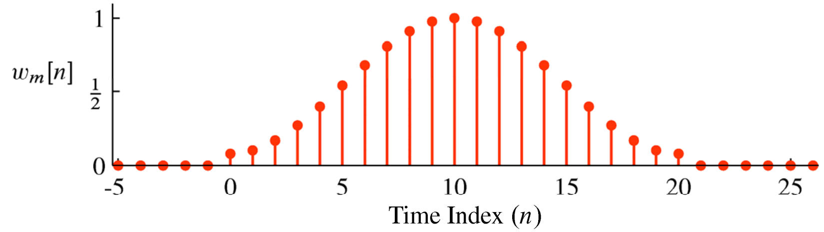
$$H_L(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} w_L[n]h[n]e^{-j\hat{\omega}n} \rightarrow \sum_{n=0}^{L-1} w_L[n]h[n]e^{-j\hat{\omega}_k n}$$

$$\hat{\omega}_k = (2\pi / N)k, \quad k = 0,1,2,\ldots N-1$$

No easy DTFT. Use zero-padded DFT to get DTFT samples

Window Filter Design

- Plot of Length-21 Hamming window



Hamming Window

$$w_m[n] = \begin{cases} 0 & n < 0 \\ 0.54 - 0.46\cos(2\pi(n)/(L-1)) & 0 \le n < L \\ 0 & n \ge L \end{cases}$$

Demo of **filterdesign** GUI

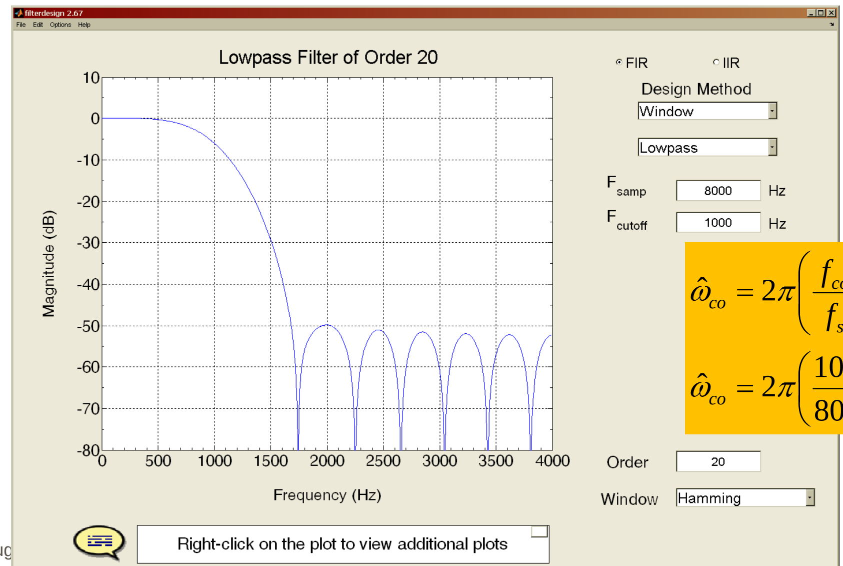
$$\hat{\omega}_{co} = 2\pi\left(\frac{f_{co}}{f_s}\right)$$

$$\hat{\omega}_{co} = 2\pi\left(\frac{0.4}{2}\right) = 0.4\pi$$

- Show filter designs in the following order:
  - Set fs=2, and cutoff freq = 0.4
  - Rectangular Window: M=20, M=40, M=200
  - Show Slide to define passband & stopband
  - Show Slide with Template for Filter Design Specs
  - Hamming Window: M=20, M=40
    - Need to reset cutoff when Window Type is changed.
  - Hamming Window for L=40 in dB (click Magnitude)
  - Hamming Window for L=40, zoom in on passband
  - Hamming Window: M=200
  - Same for von Hann?

Filter Design GUI

- Cutoff Frequency w.r.t. Sampling Rate

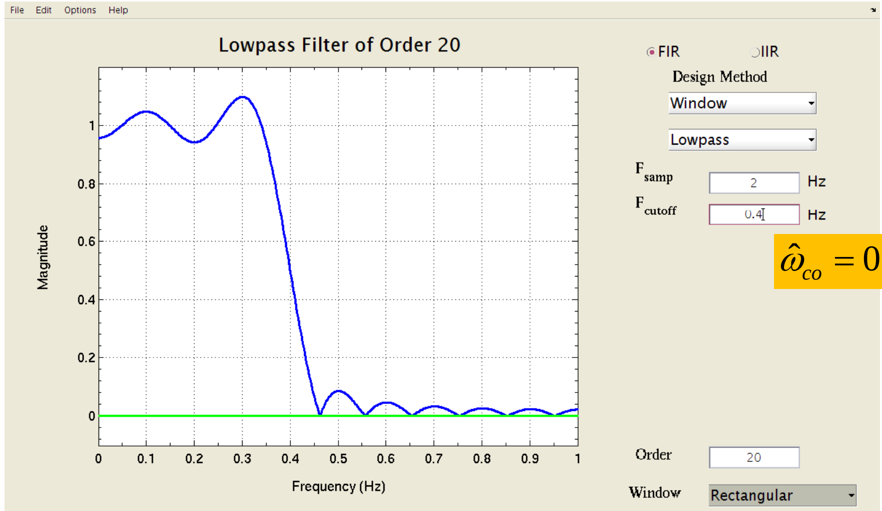


$$\hat{\omega}_{co} = 2\pi\left(\frac{f_{co}}{f_s}\right)$$

$$\hat{\omega}_{co} = 2\pi\left(\frac{1000}{8000}\right) = 0.25\pi$$

Filter Design via Rectangular Windowing (L=21)

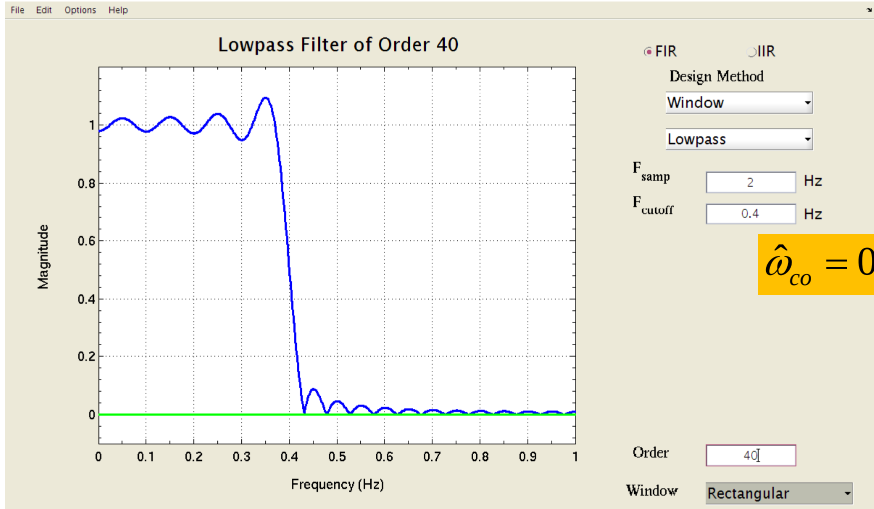
- Rectangular Window, L=21 (order M=20)



$\hat{\omega}_{co} = 0.4\pi$

Filter Design via Rectangular Windowing (L=41)

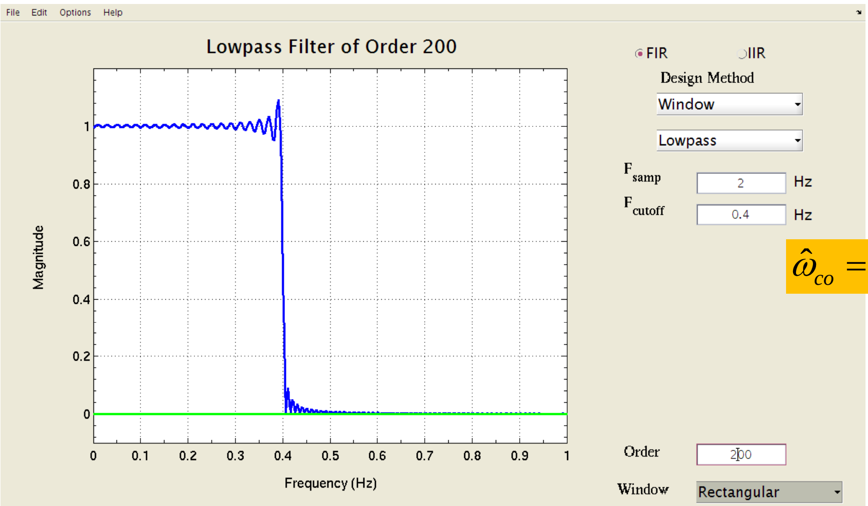
- Rectangular Window, L=41 (order M=40)



$\hat{\omega}_{co} = 0.4\pi$

Filter Design via Rectangular Windowing (L=201)

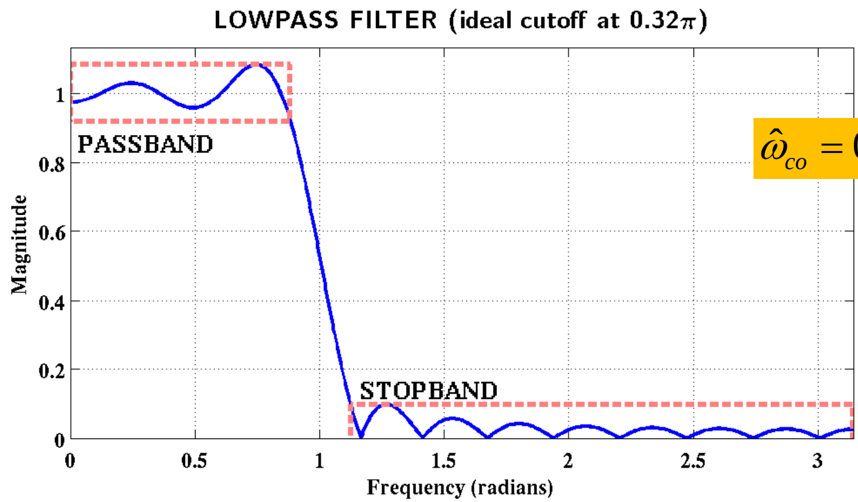
- Rectangular Window, L=201 (order M=200)



$\hat{\omega}_{co} = 0.4\pi$

Filter Design: Define Passband & Stopband

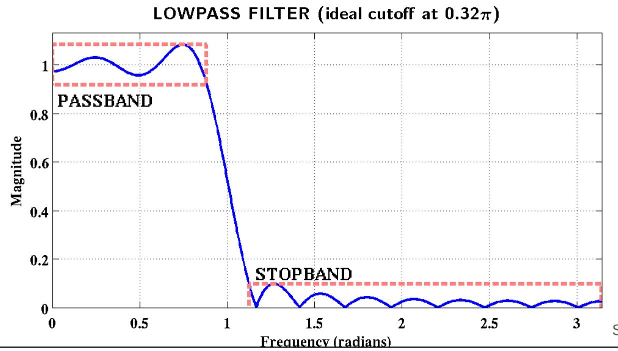
- Rectangular Window, L=41 (order M=40)



$\hat{\omega}_{co} = 0.32\pi$

Ripples, Band edges, & Transition Width

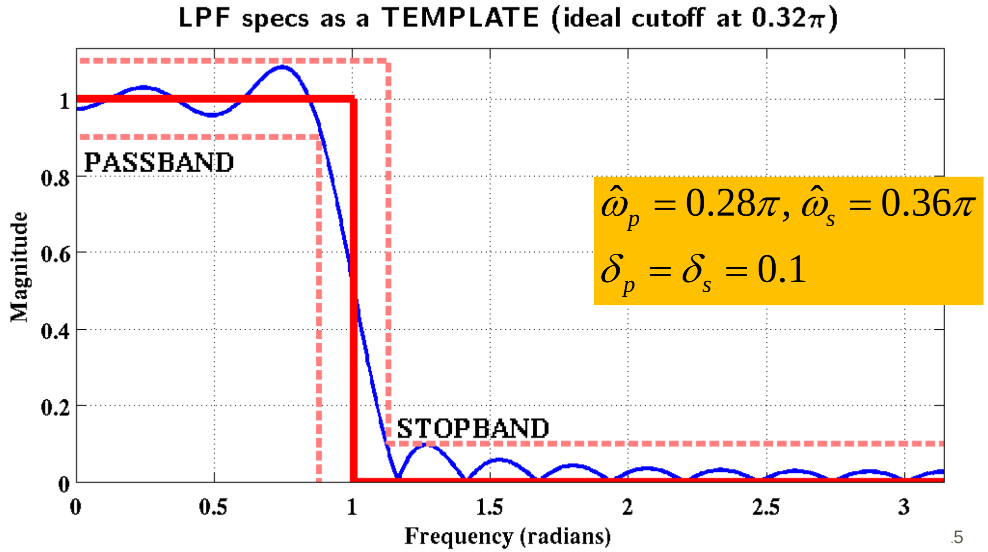
- Passband Ripple is one plus or minus δ_p
- Stopband Ripple is less than δ_s
- Band edges are $\hat{\omega}_p, \hat{\omega}_s$
- Transition Width $\Delta\omega = \hat{\omega}_s - \hat{\omega}_p$



Can't have it all: *small transition width, small ripples, and lowest possible order (M)*

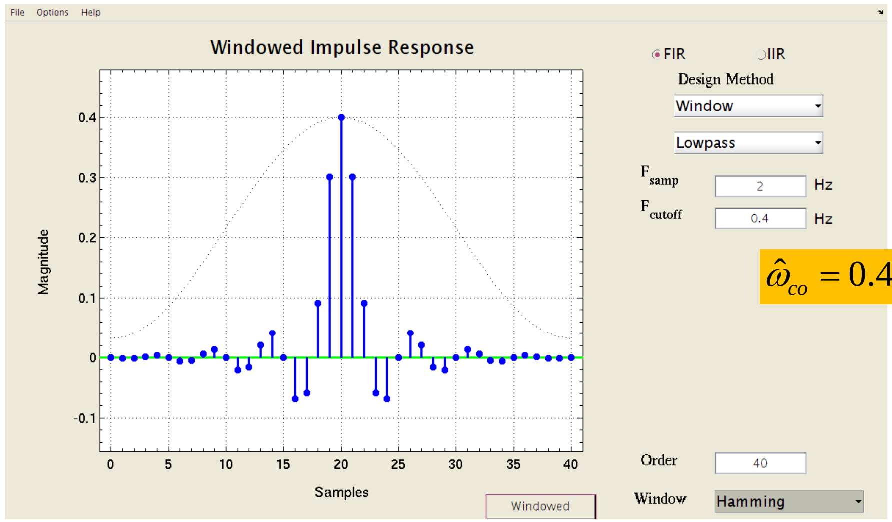
Filter Design: Tolerance Template

- Want the actual response inside the template



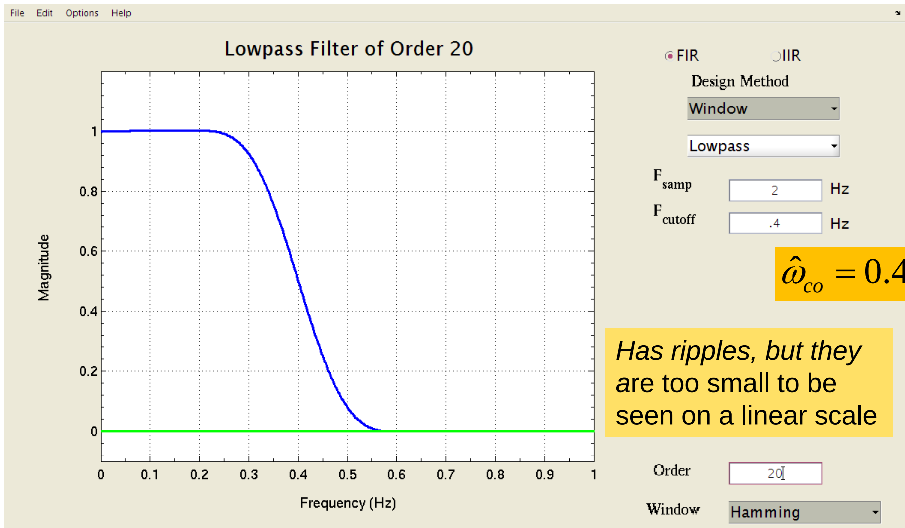
Hamming Window applied to ideal LPF impulse response

- Hamming Window, L=41 (order M=40)



Filter Design with Hamming Window (L=21)

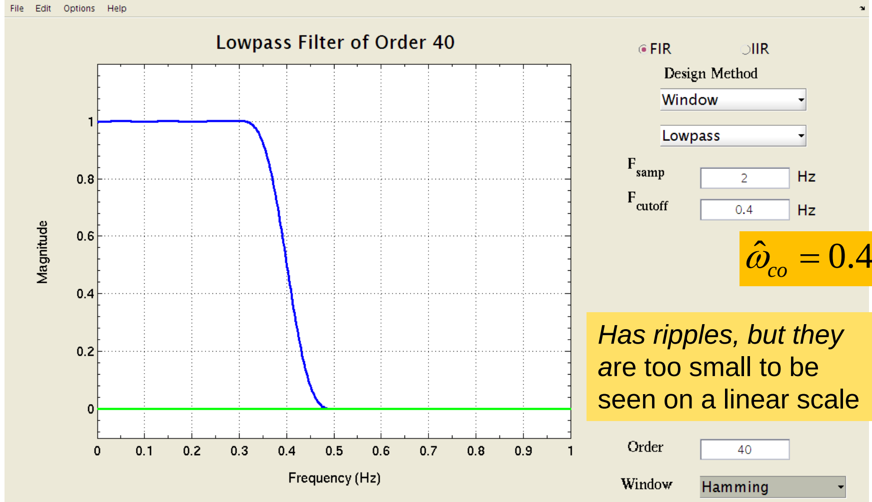
- Hamming Window, L=21 (order M=20)



Has ripples, but they are too small to be seen on a linear scale

Filter Design with Hamming Window (L=41)

- Hamming Window, L=41 (order M=40)

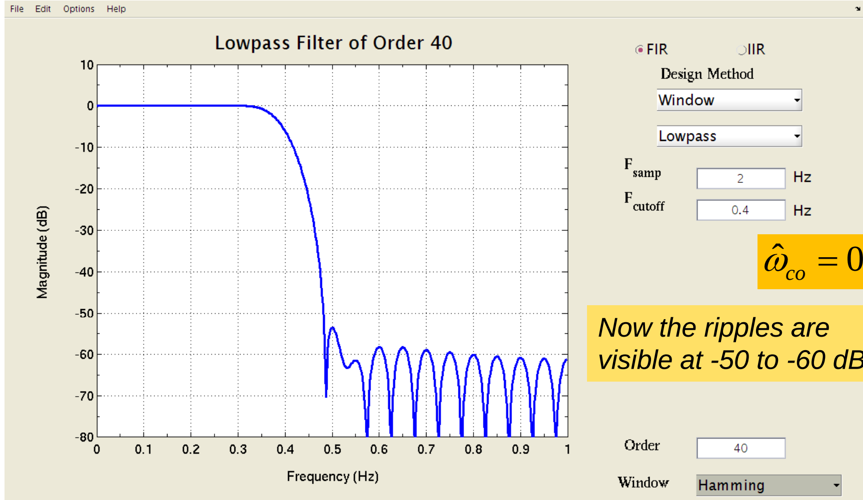


$\hat{\omega}_{co} = 0.4\pi$

Has ripples, but they are too small to be seen on a linear scale

Hamming Window LPF (L=41) Log Magnitude

- Hamming Window, L=41 (order M=40)

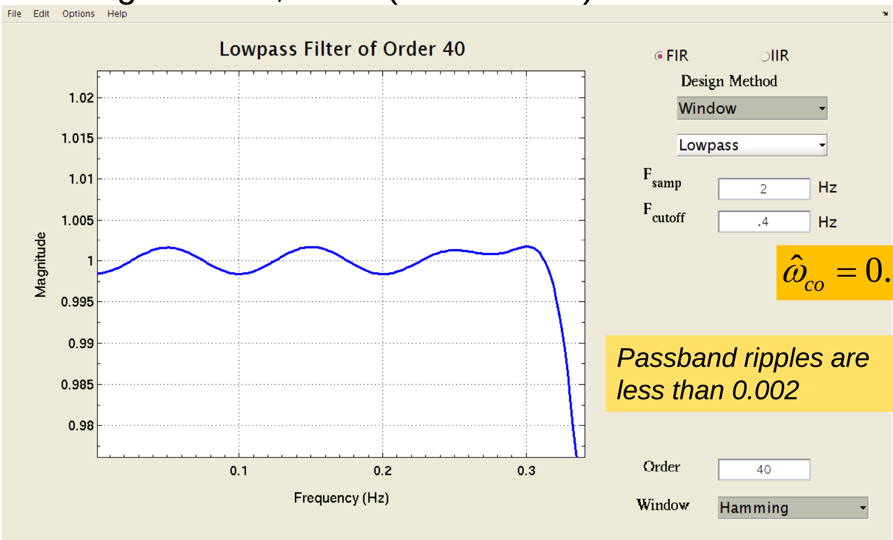


$\hat{\omega}_{co} = 0.4\pi$

Now the ripples are visible at -50 to -60 dB

Filter Design: zoom on passband ripples

- Hamming Window, L=41 (order M=40)



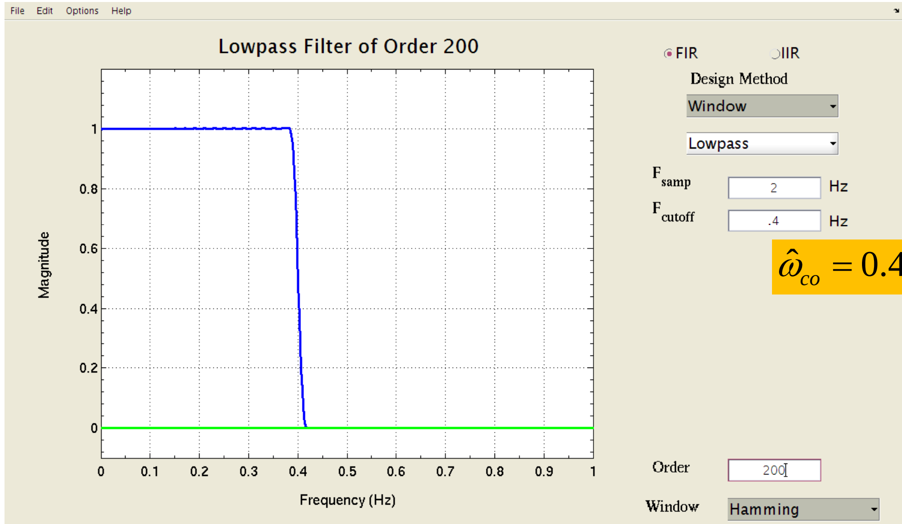
$\hat{\omega}_{co} = 0.4\pi$

Passband ripples are less than 0.002

NOTE: If Zoom is ON then Ideal Filter Lines can not be

High Order FIR Filter Design with Hamming Window

- Hamming Window, L=201 (order M=200)



$\hat{\omega}_{co} = 0.4\pi$

Hamming FIR Filter Design
Ripples and Band Edges

- Transition width is inversely proportional to L
- Ripples do not change with L
- Another window called the Kaiser window can control the ripple height
  - But passband ripple = stopband ripple
- Optimization methods such as PMFIR can control both ripples and the transition width