DSP First, 2/e

Lecture 25
Second-Order IIR Filters: 3-Domains

READING ASSIGNMENTS

- This Lecture:
  - Chapter 10, Sects. 10-11 and 10-12
- Other Reading:
  - Example of IIR LPF in Sect. 10-13
    - POLES & ZEROS
    - Frequency Response
    - Impulse Response

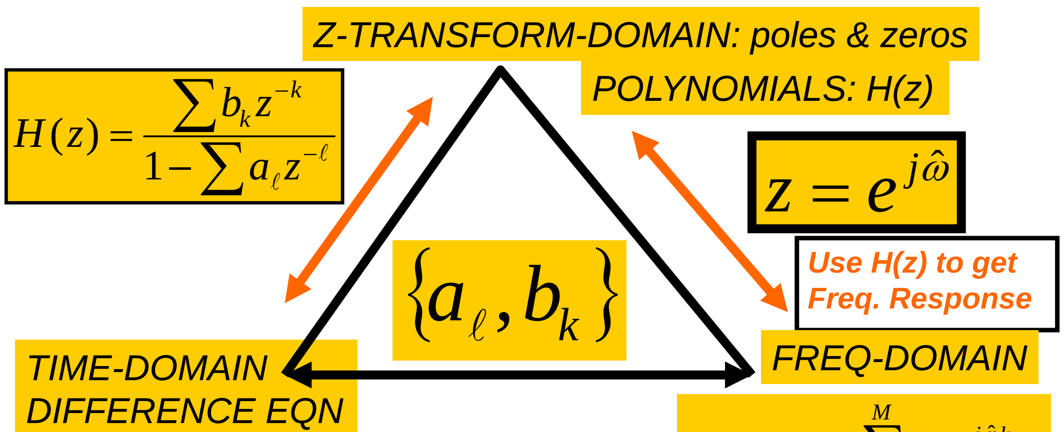
LECTURE OBJECTIVES

- SECOND-ORDER IIR FILTERS
  - TWO FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

- H(z) can have COMPLEX POLES & ZEROS
- THREE-DOMAIN APPROACH
  - BPFs have POLES NEAR THE UNIT CIRCLE

THREE DOMAINS for IIR



$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{\sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}{1 - \sum_{\ell=1}^{N} a_\ell e^{-j\hat{\omega}\ell}}$$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

MORE POLES

- Denominator is QUADRATIC
  - 2 Poles: REAL
  - or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2 + 4a_2}}{2}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{b_0 z^2 + b_1 z^1 + b_2}{z^2 - a_1 z^1 - a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

TWO COMPLEX POLES

- Find Impulse Response ?
  - Can OSCILLATE vs. n
  - "RESONANCE"

$$(p_k)^n = (re^{j\theta})^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**
  - Depends on Pole Location
  - Close to the Unit Circle?
    - Make **BANDPASS FILTER**

$$\text{pole is @ } re^{j\theta}$$
$$r \to 1?$$

Inverse z-Transform?

- *SECOND-ORDER IIR FILTERS*

$$H(z) = \frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

- *H(z) can have COMPLEX POLES & ZEROS*

$$H(z) = \frac{0.5}{1 - 0.9e^{j\pi/3}z^{-1}} + \frac{0.5}{1 - 0.9e^{-j\pi/3}z^{-1}}$$

$$2 \text{ Poles}: z = 0.9e^{\pm j\pi/3}$$

2nd ORDER EXAMPLE

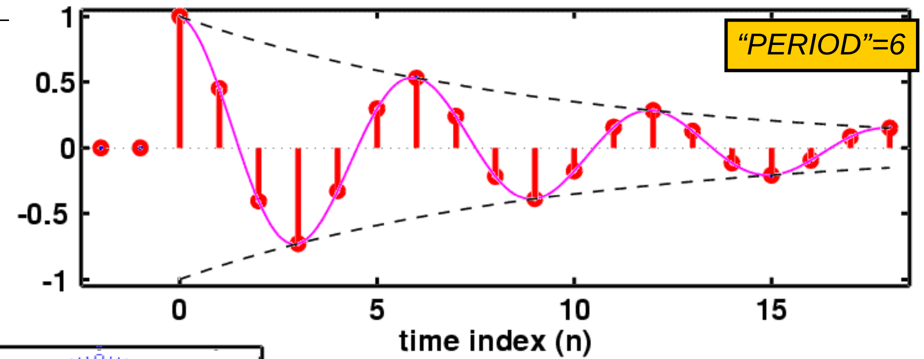
$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3} n) u[n] = (0.9)^n \tfrac{1}{2}(e^{j\pi n/3} + e^{-j\pi n/3}) u[n]$$

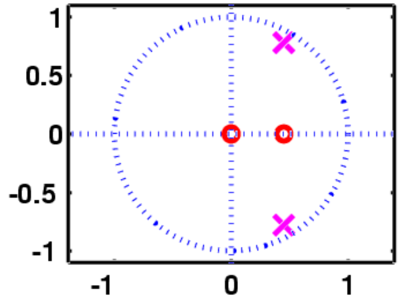
$$H(z) = \frac{0.5}{1 - 0.9e^{j\pi/3} z^{-1}} + \frac{0.5}{1 - 0.9e^{-j\pi/3} z^{-1}}$$

$$H(z) = \frac{1 - 0.9\cos(\pi/3) z^{-1}}{(1 - 0.9e^{j\pi/3} z^{-1})(1 - 0.9e^{-j\pi/3} z^{-1})}$$

$$H(z) = \frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

h[n]: Decays & Oscillates





$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3} n) u[n]$$

$$H(z) = \frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

2nd ORDER EX: n-Domain

$$H(z) = \frac{1 - 0.45 z^{-1}}{1 - 0.9 z^{-1} + 0.81 z^{-2}}$$

$$y[n] = 0.9 y[n-1] - 0.81 y[n-2] + x[n] - 0.45 x[n-1]$$

```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```


2nd ORDER Z-transform PAIRS

GENERAL ENTRY for z-Transform TABLE

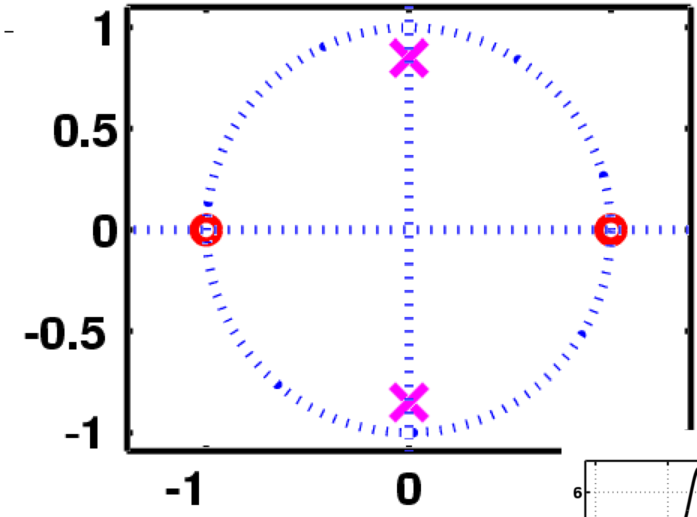
$$h[n] = r^n \cos(\theta n) u[n]$$

$$H(z) = \frac{1 - r\cos(\theta) z^{-1}}{1 - 2r\cos(\theta) z^{-1} + r^2 z^{-2}}$$

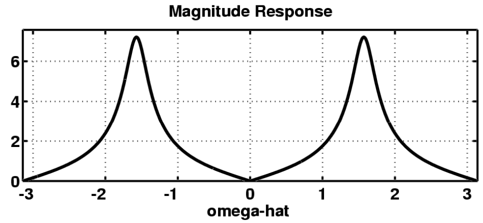
$$h[n] = A r^n \cos(\theta n + \varphi) u[n]$$

$$H(z) = \frac{\cos(\varphi) - r\cos(\theta - \varphi) z^{-1}}{1 - 2r\cos(\theta) z^{-1} + r^2 z^{-2}}$$

Complex POLE-ZERO PLOT

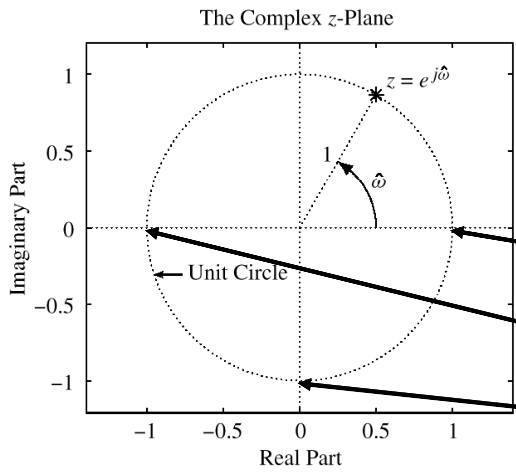


$$\frac{1-z^{-2}}{1+0.7225z^{-2}}$$



UNIT CIRCLE

- MAPPING BETWEEN z and $\hat{\omega}$



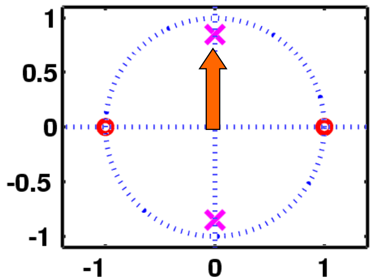
$$z = e^{j\hat{\omega}}$$

$$z = 1 \leftrightarrow \hat{\omega} = 0$$

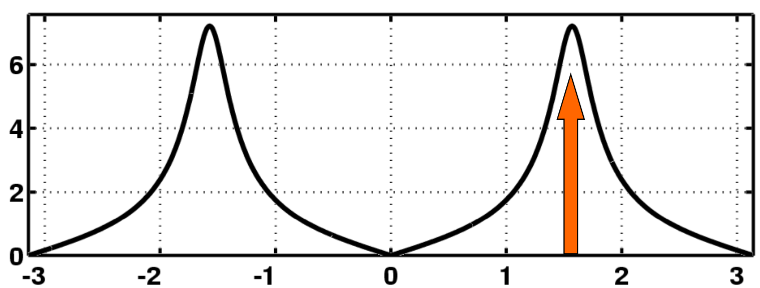
$$z = -1 \leftrightarrow \hat{\omega} = \pm\pi$$

$$z = \pm j \leftrightarrow \hat{\omega} = \pm\tfrac{1}{2}\pi$$

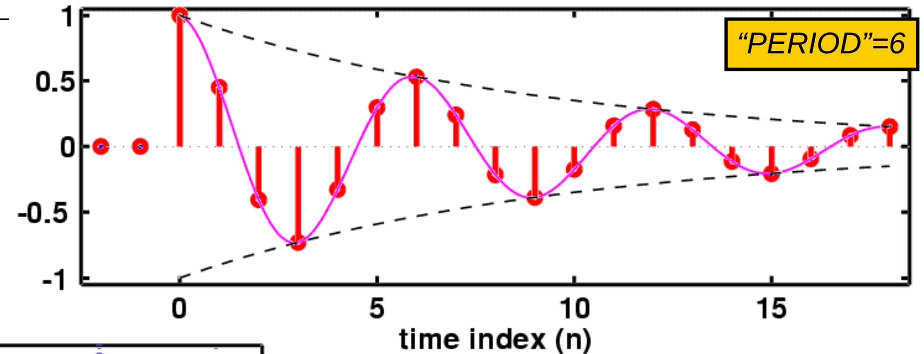
FREQUENCY RESPONSE from POLE-ZERO PLOT

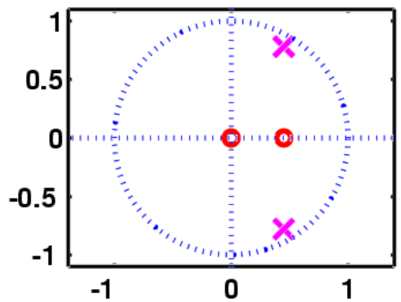


$$H(e^{j\hat{\omega}}) = \frac{1-e^{-j2\hat{\omega}}}{1+0.7225e^{-j2\hat{\omega}}}$$



h[n]: Decays & Oscillates



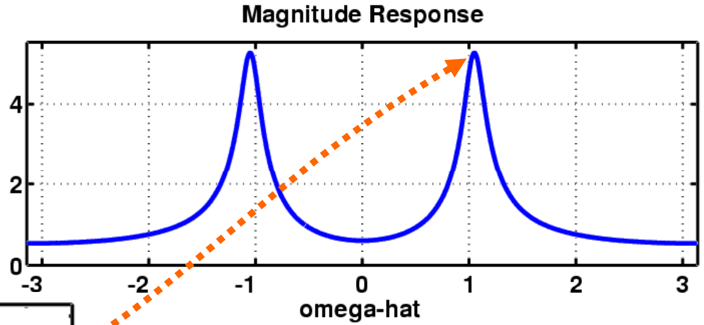


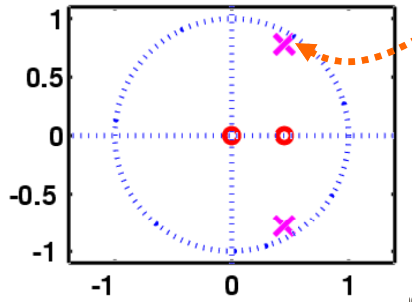
$$h[n] = (0.9)^n \cos\left(\tfrac{\pi}{3}n\right)u[n]$$

$$H(z) = \frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

Complex POLE-ZERO PLOT

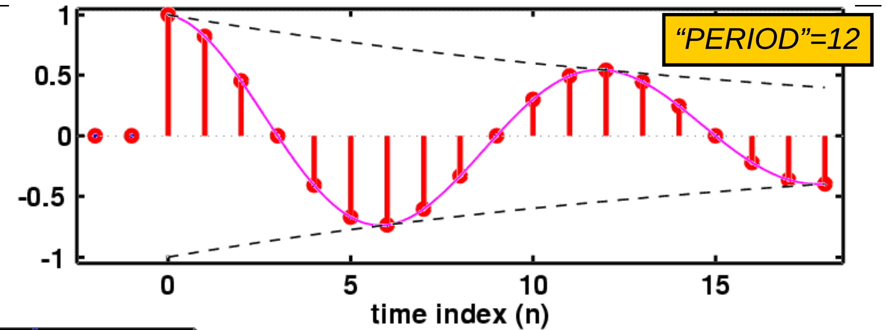
Where is the peak?

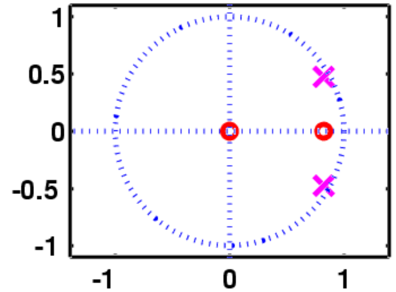




$$H(z)=\frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

h[n]: Decays & Oscillates



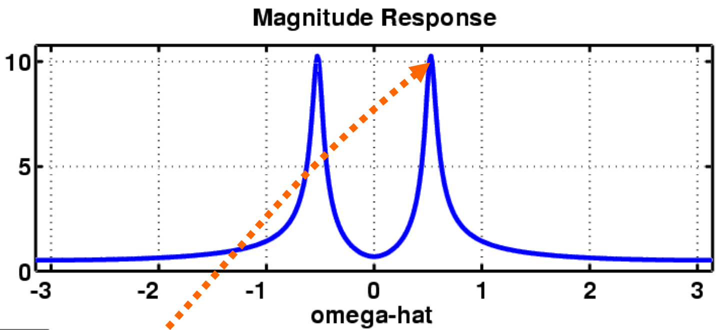


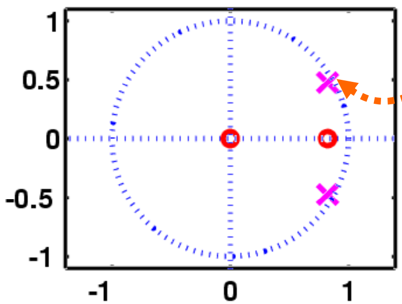
$$h[n]=(0.95)^n\cos\left(\tfrac{\pi}{6}n\right)u[n]$$

$$\frac{1-0.8227z^{-1}}{1-1.6454z^{-1}+0.9025z^{-2}}$$

Complex POLE-ZERO PLOT

Where is the peak?

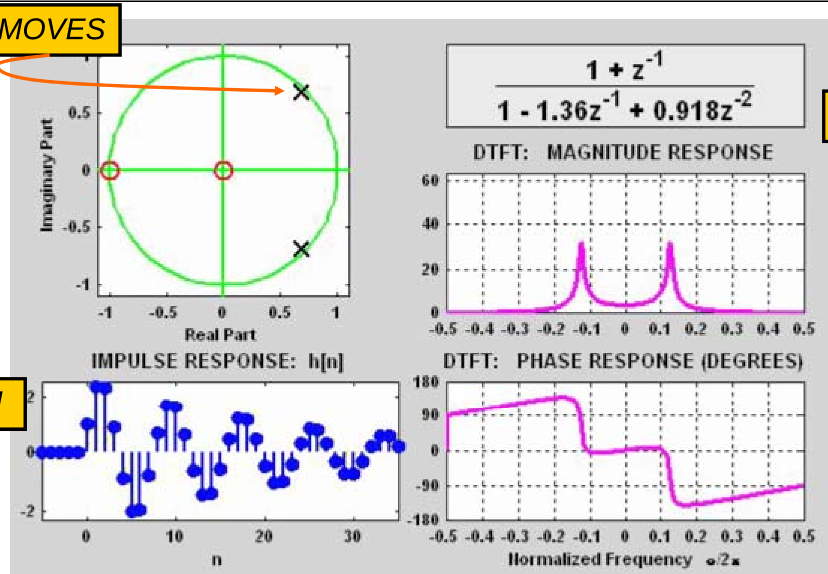




$$\frac{1-0.8227z^{-1}}{1-1.6454z^{-1}+0.9025z^{-2}}$$

3 DOMAINS MOVIE: IIR

POLE MOVES



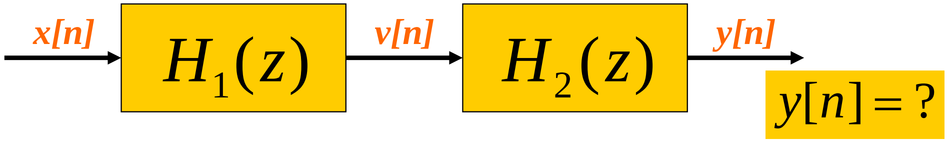
H(z)

H(ω)

h[n]

CASCADE: Pole-Zero Cancellation

- Multiply the z-transforms



$$v[n] = x[n] + 0.5x[n-1] - 0.5x[n-2]$$

$$H_2(z) = 1 - z^{-1}$$

$$x[n] = u[n] + (0.5)^n u[n]$$

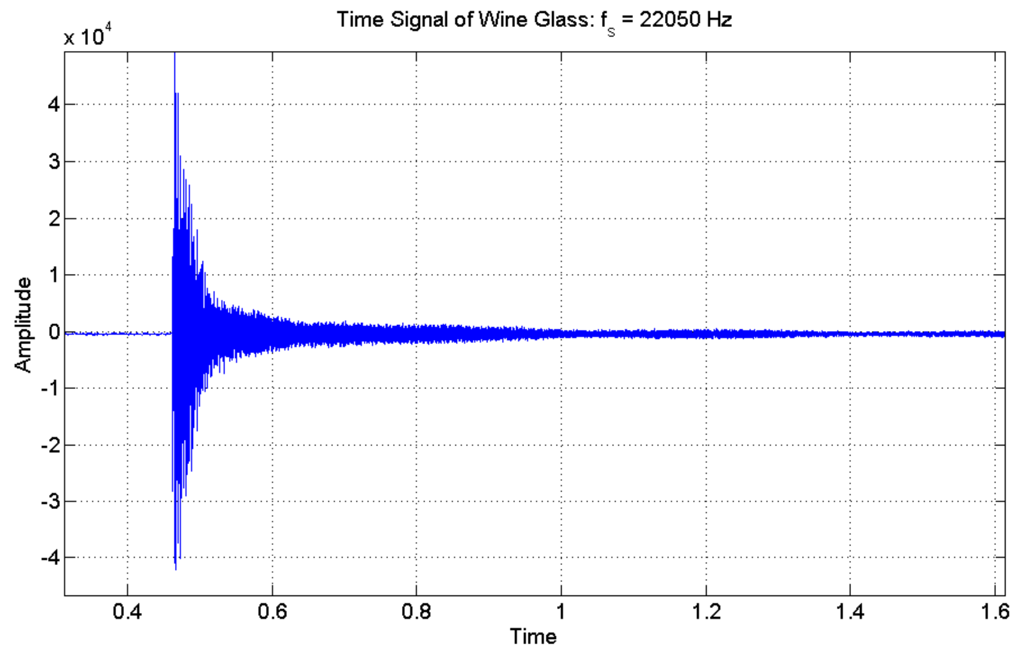
YouTube Links: h(t)

- Follow some links, and see h(t) in action
  - ---------------
- Maf Lewis films - Voice Breaks Glass w/ Tara Busch
- http://www.youtube.com/watch?v=amuPoPkAlx8&feature=related
  - ---------------
  - Show the video above first, then the “lab measurements” below
  - ---------------
- **How to break a wine glass with sound – Lab Measurements**
- http://www.youtube.com/watch?v=JiM6AtNLXX4&feature=related
  - -----------------------
- Indian Temple (Demo of changing frequency content in an image):
- http://www.youtube.com/watch?v=EaRj0q3YoGM&feature=related
  - --------------------------

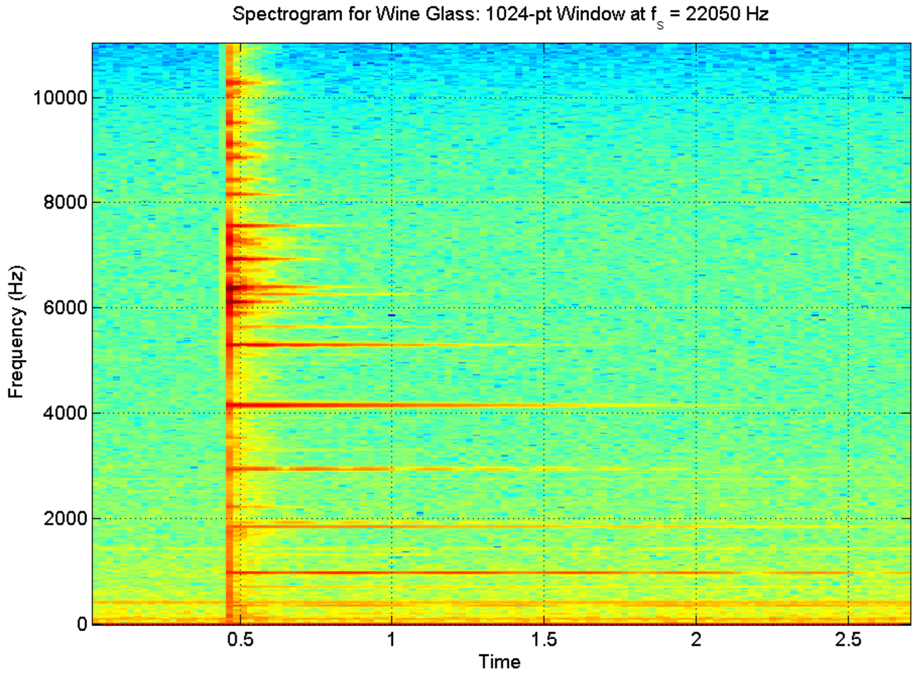
Ping, Ring & Sing

- Breaking a Wine Glass by Singing
- Ping
  - What is the input?
- Ring
  - What are you measuring?
- Thinking (Processing)
- Sing
  - What is the input?

Time Domain (then FFT)



Fourier Domain (FFT)



Recall Frequency Response

- <u>**Sinusoid-in gives sinusoid-out**</u>
  - True for LTI systems
  - Seems to require an infinite-length sinusoid

$$x[n] = \cos(\hat{\omega}_0 n) \quad \text{for } -\infty < n < \infty$$

  - Output is

$$y[n] = H(e^{j\hat{\omega}_0})\cos(\hat{\omega}_0 n) \quad \text{for } -\infty < n < \infty$$

  - Similar for real-world sinusoids starting at n=0

$$x[n] = \cos(\hat{\omega}_0 n)u[n] = \begin{cases} \cos(\hat{\omega}_0 n) & n \geq 0 \\ 0 & n < 0 \end{cases}$$