PROBLEM:

The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - e^{j\pi/3}e^{-j\hat{\omega}})(1 - e^{-j\pi/3}e^{-j\hat{\omega}}). \tag{1}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \leq \hat{\omega} \leq \pi$ will $y[n] = 0$ for all n?

(d) The frequency response in Equation (1) is written as a product of factors suggesting that it could be implemented as a cascade of several systems. By suitably grouping the factors and multiplying them together, obtain a representation as the cascade of *two* systems each of which has only *real* filter coefficients. Give the frequency responses and impulse responses of the two systems and draw a block diagram of the cascade system.